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FUZZY OPTIMIZATION MODEL OF PROJECT NETWORK WITH UNCERTAINTY

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ABSTRACT

Cost-time trade-off is a major problem in project network analysis because reducing project duration generally increases project costs, while minimizing costs may extend project completion time. Existing studies on project crashing and multi-objective optimization mainly focus on deterministic optimization or represent uncertainty using fuzzy input parameters, which often increase computational complexity. This study proposes a Fuzzy Multi-Objective Linear Programming (FMOLP) model based on Zimmermann's fuzzy goal programming framework to obtain a balanced compromise solution between project duration and cost. Unlike approaches that model fuzzy activity durations or costs, the proposed model represents fuzziness through decision-maker satisfaction levels using linear membership functions constructed from the best and worst values of each objective. The model is applied to a project network optimization problem with two objectives: minimizing project cost and minimizing project duration. Individual optimization produces extreme solutions, namely a minimum cost of Rp. 142.226.975 with a duration of 156 days and a minimum duration of 135 days with a total cost of Rp. 146.480.100. The proposed FMOLP model generates a compromise solution with a project duration of 141 days and a total cost of Rp. 143.243.125, including an additional acceleration cost of Rp. 1.016.150. These findings demonstrate that the proposed approach provides a computationally simple and practical framework for balancing conflicting project objectives under uncertainty.

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1. INTRODUCTION

A project consists of a set of interrelated activities that must be executed in a specific sequence, where each activity requires both time and cost [1], [2]. In project management, one of the most important issues is the cost-time trade-off (CTT), in which reducing project duration generally requires additional resources and increases project costs, while minimizing costs often prolongs project completion time [3], [4]. This conflicting relationship creates difficulty for decision makers in determining an appropriate balance between project duration and cost, particularly in project crashing problems where activities are accelerated at additional costs [5]. Therefore, balancing project duration and project cost has become a critical problem in project network analysis, especially in projects vulnerable to delays and budget overruns [6], [7].

Traditional project scheduling methods such as the Critical Path Method (CPM) and Program Evaluation and Review Technique (PERT) are widely used for planning and controlling project activities [8], [9]. However, conventional CPM and PERT are primarily deterministic and mainly focus on scheduling analysis rather than simultaneous optimization of conflicting objectives [10]. To address this limitation, several studies have extended these methods into fuzzy CPM and fuzzy PERT frameworks by representing activity durations or costs as fuzzy numbers [11], [12]. Although these approaches are capable of modeling parameter uncertainty more realistically, they generally increase computational complexity and often require more complicated fuzzy arithmetic operations [13].

In addition to fuzzy CPM and fuzzy PERT, multi-objective optimization approaches such as goal programming and metaheuristic algorithms have also been applied to solve cost-time trade-off problems [14], [15]. Goal programming methods are effective in handling multiple objectives, but they commonly depend on predetermined target priorities and may produce solutions that are highly sensitive to the assigned weights [16]. Meanwhile, metaheuristic approaches such as genetic algorithms and evolutionary optimization are flexible in exploring large solution spaces, but they often require extensive parameter tuning, high computational effort, and do not always provide transparent decision-making interpretations [17]. Consequently, there remains a need for an optimization framework that is computationally simpler, interpretable, and capable of incorporating decision-maker preferences in balancing project duration and cost.

Fuzzy Multi-Objective Linear Programming (FMOLP) has emerged as a promising alternative for handling conflicting objectives under uncertainty [18]. However, previous FMOLP studies in project optimization mainly focus on fuzzy input parameters, such as fuzzy activity durations and fuzzy costs [19]. In contrast, relatively limited attention has been given to representing uncertainty through fuzzy aspiration levels or decision-maker satisfaction levels within project network analysis [20]. As a result, the scientific positioning of FMOLP in project crashing problems remains insufficiently explored, particularly regarding how compromise solutions can be obtained while preserving the simplicity of linear programming models.

Unlike previous studies, this research does not model uncertainty through fuzzy activity durations or fuzzy cost coefficients. Instead, uncertainty is represented through decision-maker satisfaction levels using linear membership functions constructed from the best and worst values of each objective. This approach follows Zimmermann's fuzzy goal programming framework and enables the model to maintain the computational tractability of deterministic linear programming while still incorporating subjective decision preferences [21].

The novelty of this study lies in three aspects. First, this study integrates project network crashing problems with FMOLP using fuzzy aspiration levels rather than fuzzy input parameters. Second, the proposed model provides a simpler and more interpretable alternative compared with fuzzy CPM, fuzzy PERT, and metaheuristic optimization approaches, because it avoids complex fuzzy arithmetic and extensive heuristic parameter tuning [22]. Third, the model generates a compromise solution between project duration and project cost through satisfaction-based optimization while preserving computational efficiency.

Accordingly, the theoretical contribution of this study is the extension of FMOLP application in project network analysis through satisfaction-based uncertainty modeling. Methodologically, this study develops a linear membership-based FMOLP framework for solving cost-time trade-off problems in project crashing. Practically, the proposed model provides project managers with a computationally efficient and easily interpretable decision-support tool for balancing project duration and cost under conflicting objectives [23].

2. RESEARCH METHOD

This study develops a cost-time trade-off model in project network analysis using the Fuzzy Multi-Objective Linear Programming (FMOLP) approach [24]. The modeling process begins by formulating two objective functions, namely minimizing project cost and minimizing project completion time, within a linear programming framework based on the project network structure [25].

In this study, uncertainty is represented through fuzzy aspiration levels rather than fuzzy input parameters. Specifically, the best (Z_i^*) and worst (Z_i^0) values for each objective function are obtained by solving each objective individually using deterministic linear programming. These values are then used as reference points to construct linear membership functions that represent the decision maker's degree of satisfaction for each objective [26].

The membership function for each objective is defined as a piecewise linear function, where the satisfaction level increases as the objective value approaches its best value. The multi-objective problem is then transformed into a single-objective optimization model by maximizing the minimum satisfaction level (λ), subject to the system constraints and the membership function constraints [27].

The resulting FMOLP model is expressed as a standard linear programming problem with auxiliary variables and solved using a linear programming solver [28]. The solution provides a compromise optimal result that balances cost and time objectives simultaneously.

Finally, the obtained solution is compared with the individual optimization results to analyze the trade-off behavior between cost and time and to evaluate the effectiveness of the proposed FMOLP model [29].

2.1. Fuzzy Multi-objective Linear Programming (FMOLP)

Fuzzy Multi-Objective Linear Programming (FMOLP) is an optimization approach designed to address decision-making problems involving conflicting objectives under uncertainty. In this study, the FMOLP framework is applied to a project network problem using a case-based dataset derived from a standard project scheduling structure, where activity durations, acceleration limits, and cost parameters are defined based on commonly used project network assumptions in the literature [30].

Unlike classical multi-objective optimization, FMOLP incorporates uncertainty through fuzzy aspiration levels represented by membership functions. In this research, uncertainty is not modeled through fuzzy input parameters, but through the decision maker's satisfaction levels based on best and worst values obtained from deterministic optimization results [30].

In the context of project network analysis, FMOLP is used to transform each objective function minimizing cost and minimizing project duration into corresponding fuzzy membership functions. These functions are then aggregated into a single optimization model to obtain a compromise solution that balances both objectives.

To evaluate the effectiveness of the proposed model, the FMOLP results are compared with the solutions obtained from single-objective optimization, namely cost minimization and time minimization. This comparison highlights the trade-off behavior between objectives and demonstrates the advantage of FMOLP in providing a balanced solution. However, the model is applied to a single case study and does not include validation using multiple datasets or benchmarking against other fuzzy-based approaches, which may be considered for future research.

2.2. Data Analysis Stages

The steps for formulating the Fuzzy Multi-Objective Linear Programming (FMOLP) method in project network analysis in this study are as follows:

a. Model multiple objective problems and solve each goal as a linear program problem.

   The model variables used to formulate a network are as follows:

   D_{ij} : Normal activity time on *node* $i-j$

   d_{ij} : accelerated activity time on *node* $i-j$

   m_{ij} : maximum reduction of activity time on *node* $i-j$

   C_{ij} : activity costs on *node* $i-j$ within the normal time

   c_{ij} : activity costs at *node* $i-j$ with acceleration

   $C_{ij'}$: additional costs of acceleration for activity on *node* $i-j$

   x_i : time on activity *node-i*

   y_{ij} : the acceleration time for optimal activity on the *node* $i-j$

   The objective function that can be developed for the project network are:

   1) Minimize Project Cost

$$\text{Min } Z_1 = \sum C_{ij} \cdot y_{ij} \tag{1}$$

   2) Minimize total project time

$$\text{Min } Z_2 = x_{\text{last}} - x_{\text{first}} \tag{2}$$

The constraints that can be formulated on a network into a linear programming model are as follows :

1) Acceleration time for each activity (y_{ij}) must be smaller than the maximum reduction in activity time (m_{ij}).
2) The time between events is at least the same as the time of normal activity (D_{ij}) less acceleration time of activity $i - j$ (y_{ij}).

b. Determine the corresponding values for each goal at each completion obtained.

c. Establish the highest level of achievement or best value as Z_i' and the lowest achievement score or the worst score as Z_i^0 for each purpose.

d. Form a model *fuzzy* The membership function for each destination is denoted as $\mu_i(x)$ where for the problem of maximization the magnitude is:

$$\mu_i(x) = \begin{cases} 0, & \text{if } Z_i(x) \leq Z_i^0 \\ \dfrac{Z_i(x) - Z_i^0}{Z_i' - Z_i^0}, & \text{if } Z_i' < Z_i(x) < Z_i^0 \\ 1, & Z_i' < Z_i(x) < Z_i^0 \end{cases} \tag{3}$$

As for the goal minimization problem, the size of the membership function is:

$$\mu_i(x) = \begin{cases} 0, & \text{if } Z_i(x) \geq Z_i^0 \\ \dfrac{Z_i(x) - Z_i^0}{Z_i' - Z_i^0}, & \text{if } Z_i' < Z_i(x) < Z_i \\ 1, & \text{if } Z_i(x) \leq Z_i' \end{cases} \tag{4}$$

e. Change the model fuzzy to the model Fuzzy Multi Objektif Linier Programming

$$Maks \{\min \{\mu_i (x) ; i = 1, 2, \ldots k\}\} \tag{5}$$

With obstacles :

$$Ax \leq b; \; x \geq 0 \tag{6}$$

3. RESULT AND ANALYSIS

3.1. Project Network and Problem Formulation

As a case study, a project network consisting of a number of interrelated activities is used, as shown in Figure 1.

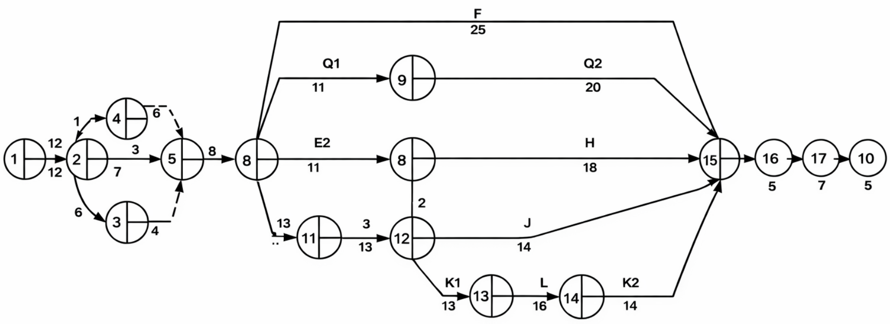

Figure 1. Project work network

The project network in Figure 1 is analyzed by considering two objective functions with different optimization directions. The first objective is to minimize project acceleration costs, while the second objective is to minimize project implementation time. The difference in the direction of these objectives indicates a conflict in decision-making, thus requiring a multi-objective optimization method capable of simultaneously addressing these conditions.

3.2. Model Multi Objective Linear Programming (MOLP)

After the project network is presented in Figure 1, the stages of solving the multi-objective optimization problem are carried out systematically as follows.

a. The first objective is to minimize the project acceleration costs (Z_1) :

$$Z_1 = \sum C_{ij} . y_{ij}$$

$$\begin{aligned} Z_1 = {} & 106.525\, y_{1.2} + 144.100\, y_{2.3} + 209.600\, y_{3.4} + 248.800\, y_{3.5} + 89.250\, y_{7.8} \\ & + 228.500\, y_{8.10} + 69.300\, y_{8.15} + 79.200\, y_{8.9} + 222.975\, y_{9.15} + 164.600\, y_{10.15} \\ & + 206.600\, y_{8.11} + 138.300\, y_{11.12} + 15.500\, y_{12.15} + 25.000\, y_{12.13} + 147.700\, y_{14.15} \\ & + 48.600\, y_{13.14} + 256.700\, y_{15.16} + 50.000\, y_{16.17} + 25.000\, y_{17.18} \end{aligned}$$

b. The second objective is to minimize the project implementation time (Z_2).

$$Z_2 = x_{18} - x_1$$

With the constraints for both objective functions being:

$y_{1.2} \leq 2$	$y_{12.15} \leq 2$	$x_9 - x_8 + y_{8.9} \geq 11$
$y_{2.3} \leq 1$	$y_{1.13} \leq 2$	$x_{10} - x_8 + y_{8.10} \geq 20$
$y_{3.4} \leq 1$	$y_{14.15} \leq 2$	$x_{11} - x_8 + y_{8.11} \geq 12$
$y_{3.6} \leq 1$	$y_{13.14} \leq 1$	$x_{12} - x_{11} + y_{11.12} \geq 16$
$y_{3.6} \leq 1$	$y_{15.16} \leq 2$	$x_{13} - x_{12} + y_{12.13} \geq 18$
$y_{7.4} \leq 0$	$y_{16.17} \leq 3$	$x_{14} - x_{13} + y_{13.14} \geq 10$
$y_{7.5} \leq 0$	$y_{17.18} \leq 2$	$x_{15} - x_8 + y_{8.15} \geq 25$
$y_{7.6} \leq 0$	$x_2 - x_1 + y_{1.2} \geq 12$	$x_{15} - x_9 + y_{9.15} \geq 20$
$y_{7.8} \leq 2$	$x_3 - x_2 + y_{2.3} \geq 7$	$x_{15} - x_{10} + y_{10.15} \geq 18$
$y_{8.10} \leq 3$	$x_4 - x_3 + y_{3.4} \geq 7$	$x_{15} - x_{12} + y_{12.15} \geq 18$
$y_{8.9} \leq 1$	$x_5 - x_3 + y_{3.5} \geq 7$	$x_{15} - x_{14} + y_{14.15} \geq 12$
$y_{9.15} \leq 1$	$x_6 - x_3 + y_{3.6} \geq 11$	$x_{16} - x_{15} + y_{15.16} \geq 15$
$y_{10.15} \leq 2$	$x_7 - x_5 + y_{5.7} \geq 0$	$x_{18} - x_{17} + y_{17.18} \geq 10$
$y_{8.11} \leq 2$	$x_7 - x_6 + y_{6.7} \geq 0$	
$y_{11.12} \leq 1$	$x_8 - x_7 + y_{7.8} \geq 18$	

With non-negative constraints:

$$y_{ij} \geq 0$$

$$x_i \geq 0;\ i, j = 1, 2, 3, \ldots$$

Based on the results of solving each objective function, two conditions were obtained in the project network optimization problem. In the first objective function, which is to minimize project implementation costs, an optimal solution was obtained without accelerating the project completion time. In this condition, the project can be completed in 156 days with a total cost of Rp. 142.226.975. This result shows that time acceleration is not necessary because it would significantly increase project costs without providing any benefits in the context of cost minimization.

Next, in the second objective function that focuses on minimizing project implementation time, maximum activity acceleration is carried out so that the project completion time can be shortened to 135 days. However, this acceleration impacts the increase in project costs, with total costs reaching Rp. 146.480.100. This condition indicates a trade-off between time and cost, where reducing the project duration requires the allocation of additional resources, leading to increased costs.

Based on these two results, the highest achievement level is set as the best value (Z_i') and the lowest achievement level as the worst value (Z_i^0) for each objective function. The determination of the values Z_i' and Z_i^0 is necessary as a basis for forming the membership function and adjusting the dual-objective problem model using the Fuzzy Linear Programming approach. Therefore, to obtain a solution that can simultaneously balance the objectives of cost and time minimization, the problem is subsequently solved using the Fuzzy Multi-Objective Linear Programming (FMOLP) method.

3.3. Fuzzy Multi-Objective Linear Programming (FMOLP)

a. The highest achievement level (Z_i') and the lowest achievement level (Z_i^0) for each objective function are as follows:

In the cost objective function, the optimization model minimizes the additional acceleration cost relative to the normal project condition. Therefore, the best achievement level for the cost objective is represented by zero additional acceleration cost.

Table 1: Achievement level for each goal function

Objective	Z_i'	Z_i^0	$d_i(Z_i' - Z_i^0)$
Minimize the cost	0	4.253.125	-4.253.125
Minimize the time	135	156	-21

b. Forming a model fuzzy

$$\mu_1 = \begin{cases} 0, & \text{if } Z_1. \geq 4.243.125 \\ 0 < \mu_1 < 1, & \text{if } 0 < Z_1 < 4.253.125 \\ 1, & \text{if } Z_1 \leq 0 \end{cases}$$

$$\mu_2 = \begin{cases} 0, & \text{if } Z_2. \geq 156 \\ 0 < \mu_2 < 1, & \text{if } 135 < Z_2 < 156 \\ 1, & \text{if } Z_2 \leq 135 \end{cases}$$

c. Converting the fuzzy model to Fuzzy Multi Objective Linear Programming

1). Objective Function
Max λ

2). Contraints

$$\lambda \leq 1/-4.253.125(106.525\, y_{1.2} + 144.100\, y_{2.3} + 209.600\, y_{3.4} + 248.800\, y_{3.5} + 89.250\, y_{7.8} + 228.500\, y_{8.10} + 69.300\, y_{8.15} + 79.200\, y_{8.9} + 222.975\, y_{9.15} + 164.600\, y_{10.15} + 206.600\, y_{8.11} + 138.300\, y_{11.12} + 15.500\, y_{12.15} + 25.000\, y_{12.13} + 147.700\, y_{14.15} + 48.600\, y_{13.14} + 256.700\, y_{15.16} + 50.000\, y_{16.17} + 25.000\, y_{17.18})$$

$$\lambda \leq 1/-21\big((x_{18} - x_1) - 156\big)$$

$y_{1.2} \leq 2$, $y_{2.3} \leq 1$, $y_{3.4} \leq 1$, $y_{3.5} \leq 1$, $y_{3.6} \leq 1$, $y_{7.4} \leq 0$, $y_{7.5} \leq 0$, $y_{7.6} \leq 0$, $y_{7.8} \leq 2$, $y_{8.10} \leq 3$, $y_{8.9} \leq 1$, $y_{9.15} \leq 1$, $y_{10.15} \leq 2$, $y_{8.11} \leq 2$, $y_{11.12} \leq 1$

$y_{12.15} \leq 2$, $y_{1.13} \leq 2$, $y_{14.15} \leq 2$, $y_{13.14} \leq 1$, $y_{15.16} \leq 2$, $y_{16.17} \leq 3$, $y_{17.18} \leq 2$, $x_2 - x_1 + y_{1.2} \geq 12$, $x_3 - x_2 + y_{2.3} \geq 7$, $x_4 - x_3 + y_{3.4} \geq 7$, $x_5 - x_3 + y_{3.5} \geq 7$, $x_6 - x_3 + y_{3.6} \geq 11$, $x_7 - x_5 + y_{5.7} \geq 0$, $x_7 - x_6 + y_{6.7} \geq 0$, $x_8 - x_7 + y_{7.8} \geq 18$

$x_9 - x_8 + y_{8.9} \geq 11$, $x_{10} - x_8 + y_{8.10} \geq 20$, $x_{11} - x_8 + y_{8.11} \geq 12$, $x_{12} - x_{11} + y_{11.12} \geq 16$, $x_{13} - x_{12} + y_{12.13} \geq 18$, $x_{14} - x_{13} + y_{13.14} \geq 10$, $x_{15} - x_8 + y_{8.15} \geq 25$, $x_{15} - x_9 + y_{9.15} \geq 20$, $x_{15} - x_{10} + y_{10.15} \geq 18$, $x_{15} - x_{12} + y_{12.15} \geq 18$, $x_{15} - x_{14} + y_{14.15} \geq 12$, $x_{16} - x_{15} + y_{15.16} \geq 15$, $x_{18} - x_{17} + y_{17.18} \geq 10$

3). With non-negativity constraints
$y_{ij} \geq 0$

$x_i \geq 0; \quad i, j = 1, 2, 3, \ldots$

4). Cost function

$$\begin{aligned}\mu_{\text{cost}} &= \frac{Z_{\max} - Z}{Z_{\max} - Z_{\min}} \\ &= \frac{146,480,100 - 143,243,125}{146,480,100 - 142,226,975} \\ &= \frac{3,236,975}{4,253,125} \approx 0.76\end{aligned}$$

5). Time function

$$\begin{aligned}\mu_{\text{time}} &= \frac{T_{\max} - T}{T_{\max} - T_{\min}} \\ &= \frac{156 - 141}{156 - 135} \\ &= \frac{15}{21} \approx 0.71\end{aligned}$$

6). Value of λ
$\lambda = min(0.76, 0.71) = 0.71$

Based on the results obtained using the Fuzzy Multi-Objective Linear Programming (FMOLP) method, a compromise optimal solution is achieved that simultaneously considers both cost and time objectives. The model produces a project completion time of 141 days with a total project cost of Rp. 143.243.125, which includes an additional acceleration cost of Rp. 1.016.150 compared to the normal project cost. The optimal satisfaction level obtained is $\lambda = 0.71$, indicating the degree of compromise between the conflicting objectives.

This result lies between the extreme solutions obtained from individual optimization of each objective, namely cost minimization, which yields a total cost of Rp. 142.226.975 with a duration of 156 days, and time minimization, which results in a shorter duration of 135 days with a higher total cost of Rp. 146.480.100. These findings confirm the existence of a trade-off between project duration and cost.

The results indicate that the FMOLP approach provides a balanced alternative solution compared to single-objective optimization, where one objective is optimized at the expense of the other. By incorporating aspiration levels through membership functions, the model enables decision makers to evaluate feasible solutions within a compromise framework based on satisfaction levels.

However, this study is limited to a single project network case and does not consider variations in network structure or parameter values. In addition, the model is based on deterministic input data, where uncertainty is represented only through fuzzy aspiration levels rather than fuzzy parameters. Therefore, future research may extend this approach by incorporating fuzzy parameters, testing multiple project scenarios, and evaluating the robustness of the model under different conditions.

3.4. Results of FMOLP Optimization

The FMOLP model was implemented and solved using MATLAB R2024a on a computer equipped with an Intel Core i5 processor, 8 GB RAM, and the Windows 11 operating system. The optimization problem was formulated as a linear programming model and solved using the simplex algorithm available in the MATLAB Optimization Toolbox. The computational implementation was carried out to define the decision variables, objective functions, membership functions, and system constraints in order to obtain the optimal compromise solution between project duration and project cost.

The FMOLP model was solved to obtain a compromise solution between the conflicting objectives of minimizing project cost and project duration. The solution was derived by maximizing the minimum satisfaction level (λ) based on the defined linear membership functions. The optimization process converged successfully and produced an optimal satisfaction level of $\lambda = 0.71$, indicating a satisfactory balance between the two objectives.

The FMOLP model yields a project completion time of 141 days with a total project cost of Rp. 143.243.125, which includes an additional acceleration cost of Rp. 1.016.150 compared to the normal project cost. This result lies between the extreme solutions obtained from single-objective optimization, namely cost minimization (156 days, Rp. 142.226.975) and time minimization (135 days, Rp. 146.480.100). These findings clearly demonstrate the trade-off between project duration and cost and confirm that the FMOLP approach is capable of producing a balanced compromise solution.

Furthermore, the optimal solution determines the set of decision variables y_{ij}, which represent the selected activity accelerations contributing to the overall project optimization. These values were obtained directly from the final solution generated using the MATLAB Optimization Toolbox.

To evaluate the robustness of the proposed model, a sensitivity analysis was conducted by varying the aspiration levels of project duration and project cost by ±5% from their original values. The variations were applied independently to the upper and lower bounds used in the membership functions, while all project network constraints and activity relationships remained unchanged. For each scenario, the FMOLP model was re-solved using the same optimization procedure to evaluate the impact on project duration, total project cost, and satisfaction level (λ).

The results of the sensitivity analysis are presented in Table 2.

Table 2: Sensitivity Analysis Results Under Aspiration Level Variations

Scenario	Project Duration (days)	Total Cost (Rp.)	λ
Original model	141	143.243.125	0.71
Aspiration level -5%	142	144.015.300	0.69
Aspiration level $+5\%$	140	142.875.450	0.73

Based on Table 2, moderate variations in the aspiration levels do not significantly change the structure of the optimal solution. A decrease of 5% in the aspiration levels slightly increases project duration and total cost, whereas an increase of 5% results in a slight improvement in both objectives. In all scenarios, the satisfaction level (λ) remains relatively stable within the interval 0.69-0.73.

These results indicate that the proposed FMOLP model is relatively robust with respect to moderate changes in decision-maker preferences. The stability of the satisfaction level and optimization outcomes demonstrates that the compromise solution generated by the model is not highly sensitive to small perturbations in the aspiration parameters.

In this study, linear membership functions were employed due to their simplicity and computational efficiency. However, alternative forms such as exponential or piecewise nonlinear membership functions may provide a more flexible representation of decision-maker preferences and may be explored in future research.

Overall, these results demonstrate that the FMOLP approach effectively selects appropriate activity accelerations to balance project duration and cost while providing a stable, transparent, reproducible, and interpretable optimization framework for decision-making.

Figure 2 illustrates the relationship between aspiration level variations, project duration, and satisfaction level. The graphical representation confirms that the optimization results remain relatively stable under moderate parameter perturbations.

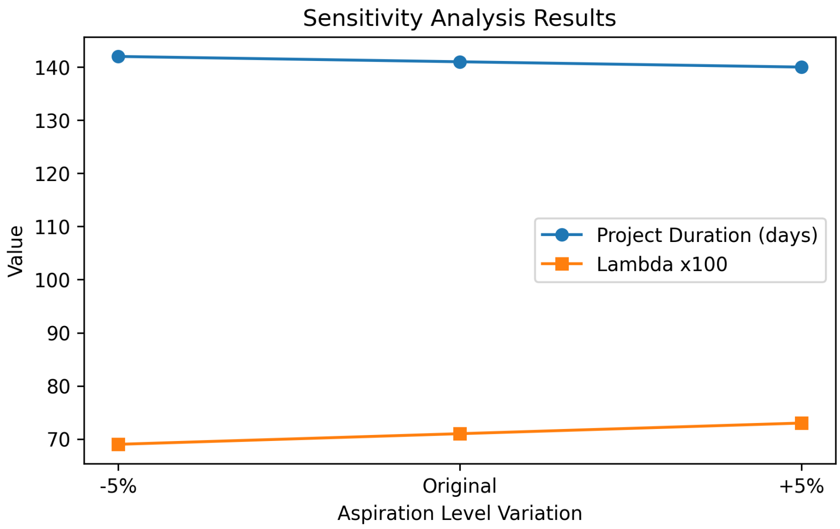

Figure 2. Sensitivity analysis results under aspiration level variations

3.5. Comparison with Single-Objective and Alternative Approaches

For comparison, solving the cost minimization model alone results in a total project cost of Rp. 142.226.975 with a project duration of 156 days, whereas solving the time minimization model produces a shorter duration of 135 days but increases the total project cost to Rp. 146.480.100. These results confirm the classical cost–time trade-off behavior in project crashing problems, where improvements in project duration are achieved at the expense of additional acceleration costs.

The proposed FMOLP model generates a compromise solution with a project duration of 141 days, a total project cost of Rp. 143.243.125, and a satisfaction level of $\lambda = 0.71$. This result indicates that the model is capable of balancing conflicting objectives while avoiding the extreme solutions produced by single-objective optimization. Compared with previous fuzzy optimization studies, particularly fuzzy CPM and fuzzy PERT approaches, the proposed model differs in the representation of uncertainty. Previous studies generally model uncertainty through fuzzy activity durations or fuzzy cost parameters, which require fuzzy arithmetic operations and may increase computational complexity. In contrast, the present study represents uncertainty through decision-maker satisfaction levels using linear membership functions based on aspiration values. This formulation preserves the structure of linear programming while still incorporating fuzzy decision preferences.

Compared with goal programming approaches, the FMOLP framework provides a more flexible compromise mechanism because the satisfaction level is determined simultaneously through membership functions rather than fixed priority targets. Furthermore, compared with metaheuristic optimization methods such as genetic algorithms, particle swarm optimization, or evolutionary approaches, the proposed model offers several computational advantages. The FMOLP formulation can be solved directly using standard linear programming solvers without iterative population-based search procedures, parameter tuning, or stochastic convergence mechanisms. As a result, the obtained solution is more transparent, reproducible, and computationally efficient for medium-scale project network problems.

From a computational perspective, the proposed FMOLP model remains tractable because the fuzzy optimization problem is transformed into a single linear programming model with additional membership constraints. This transformation avoids nonlinear fuzzy computations and reduces computational burden compared with approaches involving fuzzy numbers or metaheuristic search algorithms. However, as the number of project activities and network nodes increases, the number of decision variables and constraints also grows significantly. Consequently, large-scale project networks may require more efficient solver implementations or decomposition techniques to maintain computational efficiency. Therefore, scalability becomes an important issue for future research, particularly for multi-project scheduling environments involving resource constraints and dynamic project updates.

From a practical implementation perspective, the proposed model can be readily applied using commonly available optimization software, including spreadsheet-based solvers and standard linear programming packages. The satisfaction level ($lambda$) also provides an interpretable indicator for project managers in evaluating compromise solutions between project duration and project cost. Nevertheless, several practical challenges remain. First, determining appropriate aspiration levels and membership function boundaries depends strongly on managerial judgment and project policies. Second, real-world projects often involve uncertain resource availability, dynamic activity changes, and inter-project dependencies that are not fully represented in the current deterministic framework. Third, collecting accurate cost and acceleration data for large-scale projects may be difficult in practice.

Despite these limitations, the results demonstrate that the proposed FMOLP approach provides a computationally simple, interpretable, and practically applicable framework for solving cost time trade-off problems in project network analysis. Future research may extend this study by incorporating fuzzy activity durations and costs, integrating resource allocation and leveling constraints, and testing the robustness of the model on larger and more complex project networks.

3.6. Model Evaluation and Limitations

The results obtained from the FMOLP model are derived from the final iteration of the linear programming solver. However, detailed solver outputs and intermediate computational results are not presented in this study, as the focus is on the modeling framework and the resulting compromise solution.

In addition, the proposed model is applied to a single project network case. Although a preliminary sensitivity analysis has been conducted, the robustness of the model under multiple project network structures and large-scale project scenarios has not yet been comprehensively evaluated. Future research may include more extensive sensitivity analysis under multiple project scenarios and larger network structures to further validate the model and improve its generalizability.

4. CONCLUSION

Based on the analysis conducted, the following conclusions can be drawn:

a. The Fuzzy Multi-Objective Linear Programming (FMOLP) method provides an effective framework for supporting real-world project decision-making by enabling project managers to evaluate and balance trade-offs between project cost and duration. The use of the satisfaction level (λ) offers an intuitive measure to assess the quality of compromise solutions under multiple objectives.

b. The FMOLP approach involves formulating the cost and time objectives within a linear programming framework, determining the best and worst achievement levels, and constructing linear membership functions to represent decision-maker preferences. The resulting model is computationally tractable and can be efficiently solved using standard optimization tools, making it practical for real-world applications.

c. The results demonstrate that FMOLP produces balanced and realistic solutions compared to single-objective optimization methods, which tend to generate extreme outcomes. This highlights the capability of the approach to handle conflicting objectives in project network analysis effectively.

d. Despite its advantages, the current model is limited to deterministic project conditions and a single-project framework. Future research may focus on extending the model by integrating stochastic PERT to account for uncertainty in activity durations, developing dynamic re-scheduling mechanisms to adapt to project changes, and incorporating resource constraints as well as multi-project environments to enhance its applicability in more complex and realistic project settings.

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