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COMPARATIVE PERFORMANCE ANALYSIS OF MEWMA AND DMEWMA CONTROL CHARTS FOR PROCESS MEAN SHIFT DETECTION USING MONTE CARLO SIMULATION

Falidazia Hasanah Faizana[1], **Muhammad Mashuri**[2], **Muhammad Ahsan**[3]

[1,2,3]Department of Statistics, Institut Teknologi Sepuluh Nopember, Surabaya, Indonesia

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ABSTRACT

This study compares the performance of the Modified Exponentially Weighted Moving Average (MEWMA) and Double MEWMA (DMEWMA) control charts in detecting process mean shifts. Effective clinker quality monitoring is important in cement manufacturing because process instability may affect product quality, operational safety, and environmental conditions. Parameter calibration was conducted using Monte Carlo simulation with 20,000 replications targeting $ARL_0 \cong 370$ (approximately 370 expected in-control observations before a false alarm occurs), where the optimal parameters were selected based on the smallest ARL_1 and SDRL values. The evaluation used synthetic data under several outlier proportion scenarios and real C_4AF clinker data consisting of 100 observations. The results show that DMEWMA generally provides better out-of-control detection capability and higher accuracy than MEWMA across various scenarios. In the synthetic data scenarios considered, DMEWMA generally produced higher monitoring accuracy and out-of-control detection rates than MEWMA while maintaining low false alarm rates. Furthermore, the application to clinker data shows that DMEWMA can detect more out-of-control observations without increasing the false alarm rate. These findings indicate that the additional smoothing stage in DMEWMA improves the preservation of process shift information, resulting in more reliable process monitoring performance.

Corresponding Author:
Falidazia Hasanah Faizana,
Department of Statistics,
Institut Teknologi Sepuluh Nopember,
Email: falidazia2000@gmail.com

1. INTRODUCTION

Statistical Process Control (SPC) is a statistical method used to monitor and control a process in order to ensure that it consistently meets predetermined criteria. One of the primary tools in SPC is the control chart, which has been proven effective in monitoring process stability [1] and in detecting process shifts more rapidly, thereby minimizing the potential for product defects [2]. In industrial manufacturing, delayed detection of process instability may lead to quality deterioration, increased production costs, operational inefficiency, and economic losses. Therefore, effective process monitoring is essential to support quality assurance and maintain operational stability, particularly in continuous production industries such as cement manufacturing. In the cement industry, clinker quality control is especially critical because process conditions in the rotary kiln directly influence clinker formation and cement quality characteristics. Previous studies have shown that abnormal operating conditions in clinker production, such as unstable rotary kiln conditions and flying ash phenomena, may impair clinker quality and reduce production stability [3].

Previous occupational health studies have reported that exposure to cement dust may increase the prevalence of respiratory symptoms and respiratory disorders among cement industry workers [4], [5], [6]. Mwaiselage et al [4] found higher prevalence of chronic cough, dyspnea, chronic bronchitis, and chronic obstructive pulmonary disease (COPD) among cement factory workers exposed to cement dust. Similarly, Neghab and Choobineh [5] reported that exposed workers experienced significantly higher respiratory symptoms, abnormal chest radiographs, and reduced lung function compared with non-exposed workers. In addition, Ahmed and Abdullah [6] showed that workers in high dust concentration areas exhibited significantly higher prevalence of cough and phlegm. These findings indicate that maintaining stable clinker production and minimizing process deviations are important not only for ensuring product quality and operational stability but also for supporting occupational health and safer industrial working environments.

Control charts were first introduced by Shewhart and are recognized for their simplicity and ease of implementation. However, Shewhart control charts are generally more effective in detecting large shifts and are less sensitive to small-to-moderate process shifts. These limitations encouraged the development of memory-type control charts, such as the Cumulative Sum (CUSUM) chart introduced by Page [7] and the Exponentially Weighted Moving Average (EWMA) chart developed by Roberts [8], both of which utilize historical information from previous observations to improve sensitivity in detecting small-to-moderate process shifts. The development of EWMA-based control charts has continued in order to improve process shift detection capability. Patel and Divecha [9] proposed the Modified EWMA (MEWMA) control chart corresponding to the case ($k = 1$). Subsequently, Khan et al [10] further investigated the MEWMA structure by incorporating a correction component based on the difference between two consecutive observations and demonstrated that ($k = -\lambda/2$) minimizes the variance of the MEWMA statistic under normal distribution assumptions. Later, Alevizakos et al [11] introduced the Double Modified EWMA (DMEWMA), a two-stage control chart that repeatedly applies the MEWMA structure. The DMEWMA approach has been shown to improve detection performance, particularly for small-to-moderate process shifts.

Although previous studies have demonstrated the effectiveness of MEWMA and DMEWMA in process monitoring, comparative evaluations between these two control charts under varying contamination conditions remain limited. Most previous studies primarily focused on ARL and SDRL evaluations under simulation settings and rarely incorporated practical monitoring indicators such as false alarm rates, out-of-control detection capability, and monitoring accuracy using both synthetic and real industrial data. In addition, the monitoring performance of MEWMA and DMEWMA under different contamination proportions and process deviation levels has not been comprehensively investigated. Therefore, a broader comparative evaluation is still needed to better understand the relative monitoring performance of both control charts under realistic industrial monitoring conditions. Based on these limitations, this study aims to compare the performance of the MEWMA and DMEWMA control charts in detecting process mean shifts using both synthetic and real clinker quality data. Synthetic data were generated under three contamination proportion scenarios, namely 60:40, 80:20, and 90:10, representing different levels of process deviation severity. These scenarios were used to evaluate the sensitivity, stability, false alarm rates, out-of-control detection capability, and monitoring accuracy of the MEWMA and DMEWMA control charts under varying contamination conditions.

In addition to synthetic data, this study also employed real data consisting of Tetracalcium Aluminoferrite (C_4AF) content in the clinker production process. Tetracalcium aluminoferrite (C_4AF), commonly referred to as the ferrite phase, is one of the four principal mineral compounds formed during cement clinker production together with tricalcium silicate (C_3S), dicalcium silicate (C_2S), and tricalcium aluminate (C_3A) [12]. The C_4AF variable was selected because fluctuations in its composition may affect sulfate resistance, heat of hydration, cement setting characteristics, and overall cement quality performance [12]. In addition, instability in clinker composition may reduce production consistency and affect construction material reliability. The real dataset, obtained from Andi [13], consists of 100 observations of C_4AF levels collected from the clinker production process. In this study, the real data were used as an industrial case study to validate the monitoring performance obtained from the synthetic data evaluation and to assess the applicability of MEWMA and DMEWMA under actual production conditions.

Therefore, this study is expected to contribute to the SPC literature by providing a comprehensive comparative evaluation of the MEWMA and DMEWMA control charts under multiple contamination scenarios using both synthetic and real industrial data. The findings are expected to provide practical insights for selecting more sensitive and reliable control charts for industrial process monitoring applications, particularly in cement manufacturing industries. Based on the foregoing discussion, this study is guided by the following research questions:

RQ1. Which control chart, MEWMA or DMEWMA, provides better monitoring performance based on ARL and SDRL measures under process mean shift conditions?
RQ2. How do the MEWMA and DMEWMA control charts differ in terms of false alarm rates, out-of-control detection capability, and monitoring accuracy under various contamination proportion scenarios using synthetic data?
RQ3. Does the DMEWMA control chart demonstrate superior monitoring performance compared with MEWMA when applied to real clinker quality data under actual industrial monitoring conditions?

2. RESEARCH METHOD

This study compares two univariate control charts, namely the Modified Exponentially Weighted Moving Average (MEWMA) and Double Modified Exponentially Weighted Moving Average (DMEWMA) control charts. The MEWMA control chart employs the monitoring statistic M_t at each time point t by incorporating a combination of the current observation, the previous monitoring statistic, and the difference between the two most recent observations in order to enhance the early detection capability for small-to-moderate process shifts [10]. The MEWMA statistic at time t is defined as follows [10]:

$$M_t = \lambda x_t + (1-\lambda)M_{t-1} + k(x_t - x_{t-1}) \tag{1}$$

where M_t denotes the MEWMA statistic at time t, M_{t-1} denotes the MEWMA statistic at the previous time point $(t-1)$, X_t and X_{t-1} represent the process observations at times t and $(t-1)$, respectively, λ is the weighting parameter with $0 < \lambda \leq 1$, k is an additional parameter controlling the influence of the difference between the two most recent observations on the statistic, and the initial value is specified as $M_0 = x_0 = \mu_0$. The expectation and variance of the MEWMA statistic are given as follows:

$$E(M_t) = \mu_0 \tag{2}$$

$$\text{Var}(M_t) = \left[\frac{(\lambda + 2\lambda k + 2k^2) - \lambda(1-\lambda-k)^2(1-\lambda)^{2(t-1)}}{2-\lambda}\right]\sigma^2 \tag{3}$$

For large values of t, the asymptotic variance of the statistic M_t is expressed as:

$$\text{Var}(M_t) = \frac{(\lambda + 2\lambda k + 2k^2)}{2-\lambda}\sigma^2 \tag{4}$$

The control limits of the MEWMA control chart are defined as:

$$UCL_M = \mu_0 + L_M \sigma\sqrt{\text{Var}(M_t)} \tag{5}$$

$$LCL_M = \mu_0 - L_M \sigma\sqrt{\text{Var}(M_t)} \tag{6}$$

where σ^2 represents the process variance. The asymptotic variance of M_t reaches its minimum value when $k = -\lambda/2$; therefore, $k = -\lambda/2$ is commonly adopted as the optimal additional parameter in the MEWMA control chart [10]. The process is considered out-of-control when $M_t > UCL_M$ or $M_t < LCL_M$.

The DMEWMA control chart consists of a two-stage processing structure based on the MEWMA statistic. In other words, the monitoring statistic is generated through two layers of exponential smoothing [11]. The DMEWMA statistic is defined as follows [11]:

$$M_t = \lambda_1 x_t + (1-\lambda_1)M_{t-1} + k_1(x_t - x_{t-1}) \tag{7}$$

$$DM_t = \lambda_2 M_t + (1-\lambda_2)DM_{t-1} + k_2(M_t - M_{t-1}) \tag{8}$$

where M_t denotes the MEWMA statistic at time t, DM_t denotes the DMEWMA statistic at time t, and M_{t-1} and DM_{t-1} denote the MEWMA and DMEWMA statistics at the previous time point $(t-1)$, respectively. The parameter λ is the weighting parameter satisfying $0 < \lambda \leq 1$, whereas k represents the additional parameter. The initial values are assumed to satisfy $x_0 = M_0 = DM_0 = \mu_0$. To simplify the computational and implementation procedures, it is assumed that $k_1 = k_2 = k$ and $\lambda_1 = \lambda_2 = \lambda$. The expectation and variance of the DMEWMA statistic are:

$$E(DM_t) = \mu_0 \tag{9}$$

$$\begin{aligned}\mathrm{Var}(DM_t) = \Bigg[&(k+\lambda)^4 + 4\lambda^2(k+\lambda)^2(k+\lambda-1)^2 + 4\lambda^2(k+\lambda-1)^2(k+\lambda)^2\left[\frac{1-\theta^{t-2}}{1-\theta}\right] \\ &- 4\lambda^3(k+\lambda-1)^3(k+\lambda)(1-\lambda)\left[\frac{1-\theta^{(t-1)}}{(1-\theta)^2} - \frac{(t-1)\theta^{(t-2)}}{1-\theta}\right] \\ &+ \lambda^4(k+\lambda-1)^4\left[\frac{1+\theta-(t-1)^2\theta^{t-2}+(2t^2-6t+3)\theta^{t-1}}{(1-\theta)^3} - \frac{(t-2)^2\theta^t}{(1-\theta)^3}\right]\Bigg]\sigma^2 \end{aligned} \tag{10}$$

The asymptotic variance of the statistic DM_t, for large values of t, is expressed as follows:

$$\begin{aligned}\mathrm{Var}(DM_t) = \Bigg[&(k+\lambda)^4 + 4\lambda^2(k+\lambda)^2(k+\lambda-1)^2 + \frac{4\lambda^2(k+\lambda-1)^2(k+\lambda)^2(1-\lambda)^2}{1-\theta} \\ &- \frac{4\lambda^3(k+\lambda-1)^3(k+\lambda)(1-\lambda)}{(1-\theta)^2} + \lambda^4(k+\lambda-1)^4\left[\frac{1+\theta}{(1-\theta)^3}\right]\Bigg]\sigma^2 \end{aligned} \tag{11}$$

The control limits of the DMEWMA control chart are defined as follows:

$$UCL_{DM} = \mu_0 + L_{DM}\sigma\sqrt{\mathrm{Var}(DM_t)} \tag{12}$$

$$LCL_{DM} = \mu_0 - L_{DM}\sigma\sqrt{\mathrm{Var}(DM_t)} \tag{13}$$

where μ_0 represents the in-control process mean, σ denotes the in-control process standard deviation, and $L_{DM} > 0$ represents the control limit width constant.

Average Run Length (ARL) refers to the average number of observations required until the first out-of-control signal is detected. In evaluating control chart performance, ARL is classified into two types, namely ARL_0 and ARL_1. The value of ARL_0 represents the expected average number of observations before an observation first exceeds the control limits while the process remains in-control [14]. Meanwhile, ARL_1 represents the average number of observations required to detect an out-of-control signal when the process has actually shifted to an out-of-control condition [14]. Mathematically, ARL_0 and ARL_1 are defined as follows:

$$ARL_0 = \frac{1}{\alpha} \tag{14}$$

$$ARL_1 = \frac{1}{1-\beta} \tag{15}$$

where α denotes the Type I error probability, representing the probability of generating an out-of-control signal when the process is actually in-control, whereas β denotes the Type II error probability, representing the probability of failing to detect an out-of-control signal despite the occurrence of a process shift. In addition to ARL, another performance measure employed in this study is the Standard Deviation of Run Length (SDRL), which is used to quantify the variability of the run length distribution. SDRL provides information regarding the consistency of signal detection time that cannot be adequately explained solely by the average ARL value. Smaller SDRL values indicate that the control chart demonstrates more stable and consistent performance in detecting process shifts [15]. The SDRL statistic is defined as follows [16]:

$$SDRL = \sqrt{\frac{\sum_{i=1}^{n}(RL_i - ARL)^2}{n-1}} \tag{16}$$

where RL_i denotes the run length in the i-th simulation, ARL represents the average run length across all simulations, and n denotes the number of simulation replications.

In the implementation of the MEWMA and DMEWMA control charts, a calibration procedure is required to determine the appropriate control limit constant L for each combination of weighting parameter λ and additional parameter k. The weighting parameter values employed in this study were $\lambda = 0.25$, 0.50, and 0.75, representing low, moderate, and high smoothing levels commonly used in EWMA-based control charts [8], whereas the additional parameter values considered were $k = -\lambda/2$, $-\lambda/4$, $\lambda/4$, $\lambda/2$, λ, and 1. The evaluation across different λ values, contamination proportions, and mean shift scenarios also provides a sensitivity assessment regarding the influence of parameter variation on the detection performance of both control charts. The calibration process was conducted using Monte Carlo simulation with 20,000 replications generated from a standard normal distribution. All simulations were performed using R software with a fixed random seed (123) to ensure reproducibility of the simulation results. Monte Carlo simulation is a statistical computational method used to simulate probabilistic phenomena through the generation of random numbers based on specific probability distributions [17].

Furthermore, this method is capable of comprehensively accounting for system variability, thereby producing more realistic estimates than deterministic approaches [18]. Figure 1 presents the Monte Carlo simulation procedure for the MEWMA control chart, while Figure 2 illustrates the corresponding simulation procedure for the DMEWMA control chart used to evaluate monitoring performance based on ARL and SDRL measures.

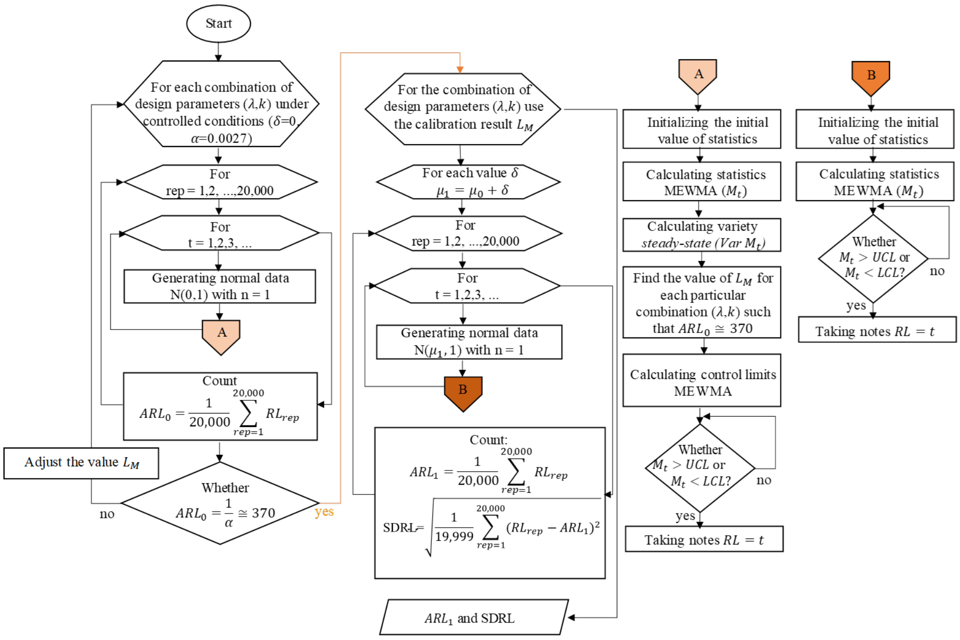


Figure 1. Monte Carlo Simulation Procedure for the MEWMA Control Chart

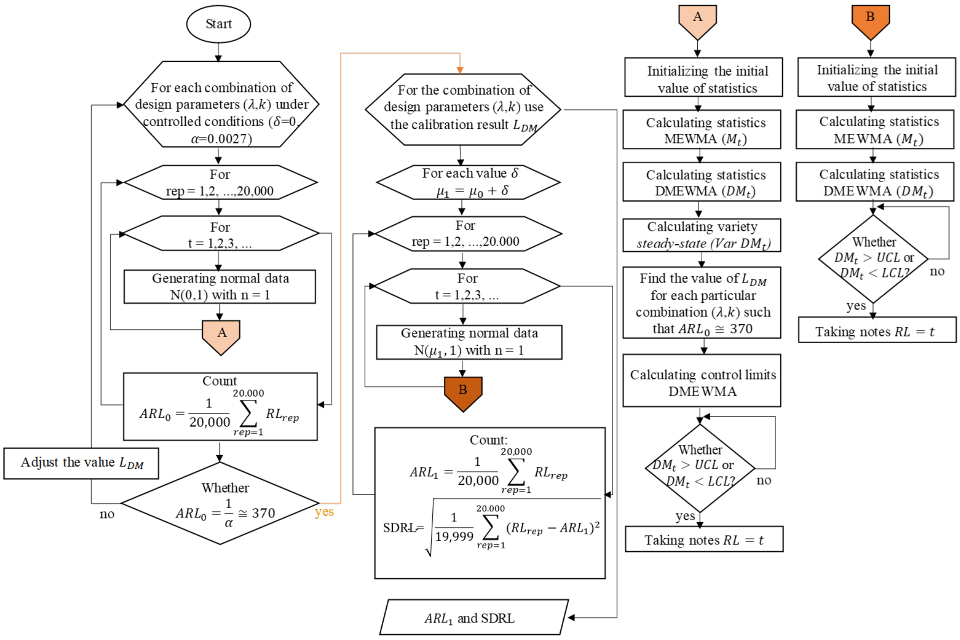


Figure 2. Monte Carlo Simulation Procedure for the DMEWMA Control Chart

Formal inferential statistical testing was not included in this study because the primary objective was to evaluate the operational monitoring performance of MEWMA and DMEWMA under controlled SPC simulation settings rather than to perform population-based statistical inference. Therefore, the analysis focused on operational performance indicators such as ARL, SDRL, false alarm rate, out-of-control detection capability, and monitoring accuracy, which are commonly used in SPC simulation studies [11]. The use of normally distributed data was intended to satisfy the assumptions underlying the MEWMA and DMEWMA control charts. The C_4AF data, collected daily from the clinker production process under routine operating conditions, were associated with the kiln process stages and obtained through laboratory-based X-Ray Fluorescence (XRF) analysis to evaluate clinker chemical composition and quality characteristics [13]. These observations were then used as sequential process measurements for quality monitoring purposes.

The real clinker quality dataset was obtained through authorized academic–industrial collaboration, and specific company information was not disclosed due to industrial confidentiality. The dataset consisted of secondary industrial monitoring data collected under actual operating conditions and was used to evaluate the practical monitoring performance of the proposed control charts. Exploratory data analysis and normality testing indicated that the C_4AF data reasonably satisfied the normality assumption underlying the implementation of the MEWMA and DMEWMA control charts, ensuring consistency with the theoretical basis of EWMA-based monitoring methods.

Nevertheless, mild departures from normality, such as slight skewness, may still influence the performance of memory-type control charts, particularly in terms of ARL, SDRL, false alarm rates, and detection sensitivity. However, previous studies have generally shown that moderate deviations from normality do not substantially reduce the performance of EWMA-based control charts unless the process distribution is heavily skewed or strongly contaminated [19]. Therefore, although the findings of this study are mainly applicable under approximately normal process conditions, the proposed evaluation framework still provides meaningful practical insights for industrial monitoring situations involving mild distributional deviations and serves as a useful benchmark for comparing the performance of MEWMA and DMEWMA under controlled monitoring conditions.

For each parameter combination (λ, k), the control limit constant L was determined iteratively until an ARL_0 value approaching 370 was obtained. Consequently, the resulting L values differed across parameter combinations because they depended on the statistical characteristics of each (λ, k) pair. Subsequently, the control chart performance was evaluated using ARL_1 and SDRL measures by simulating mean shifts of $\delta = 1.00$, 2.00, and 3.00, representing small, moderate, and large process shifts, respectively. Based on these evaluation results, the optimal parameter combination for each λ value was identified as the parameter set (λ, k, L) that produced the smallest ARL_1 and SDRL values, indicating the fastest and most stable detection capability for process shifts. These optimal parameter combinations were subsequently employed as the basis for implementing the control charts in both synthetic and real data analyses. Although statistical significance testing between the MEWMA and DMEWMA performance measures was not conducted in this study, comparative evaluation was performed descriptively using ARL, SDRL, false alarm rates, and classification accuracy measures.

In evaluating control chart performance, synthetic data were generated from a standard normal distribution, $Z_t \sim N(0, 1)$, representing the in-control condition, while the out-of-control condition was modeled by introducing a mean shift ($\delta = 3.00$) such that $X_t^* = Z_t + 3$. To assess detection performance under varying disturbance levels, the synthetic data were constructed under contamination proportion scenarios of 60:40, 80:20, and 90:10, representing high, moderate, and low contamination conditions, respectively. These scenarios were selected to provide representative monitoring conditions while maintaining manageable simulation complexity. Control chart performance was evaluated based on false alarms, out-of-control (OOC) signals, and classification accuracy in identifying process conditions.

In addition to synthetic data-based testing, the performance of the control charts was also evaluated using real data to assess the reliability of the proposed methods under actual industrial conditions. In practice, real data are generally divided into two phases, namely Phase I and Phase II. Phase I is utilized to estimate process parameters under in-control conditions, whereas Phase II is employed for monitoring activities to detect process deviations. The real dataset used in this study consisted of 100 observations, with the first 70 observations used as Phase I data and the remaining 30 observations used as Phase II data for process monitoring purposes. The separation between Phase I and Phase II data was conducted by considering the relatively homogeneous fluctuation pattern observed within specific monitoring periods [20].

Although the number of Phase II observations was relatively limited, the real-data application was intended to provide practical validation of the monitoring performance of MEWMA and DMEWMA under actual industrial conditions. Despite the shorter monitoring sequence, the real-data results remained consistent with the Monte Carlo simulation findings, where DMEWMA generally demonstrated stronger monitoring performance than MEWMA. In addition, the main evaluation of both control charts was comprehensively conducted through Monte Carlo simulation involving 20,000 replications under multiple monitoring scenarios. Therefore, the real-data analysis served as complementary industrial validation while still providing meaningful practical insight into clinker quality monitoring. Prior to implementing the control charts, exploratory data analysis (EDA), including data visualization, descriptive statistics, and distribution testing, was conducted to ensure that the assumptions underlying the control charts were satisfied.

The distribution of the C_4AF data in this study was evaluated using several normality assessment approaches, including the Kolmogorov–Smirnov (KS), Lilliefors, and Jarque–Bera tests, together with graphical evaluation using a Q–Q plot. The normality assessment was conducted only on the Phase I observations because these data were used to establish the in-control baseline and estimate the process parameters required for SPC monitoring. The KS test was selected because it compares the empirical and theoretical cumulative distribution functions [21], while the Lilliefors test is suitable for ungrouped quantitative data with mean and variance estimated directly from the observed data and can be applied to both small and large datasets [22]. In addition, the Jarque–Bera test was used because it evaluates normality based on skewness and kurtosis characteristics [23]. The Q–Q plot was also employed as a graphical diagnostic tool to visually assess the normality assumption and identify possible deviations, particularly in the tail regions of the distribution [24].

The KS test statistic is defined as follows:

$$D = \max(D^+, D^-) \tag{17}$$

where D^+ is the maximum positive difference between the empirical and theoretical cumulative distribution functions and D^- is the maximum negative difference between the empirical and theoretical cumulative distribution functions. The calculated D value is compared with the corresponding critical table value. If $D > D_{\text{tab}}$, then H_0 is rejected, indicating that the data do not follow a normal distribution. The Kolmogorov–Smirnov test is considered more appropriate for large sample sizes exceeding 40 observations and has been shown to be among the most consistent methods for detecting non-normality under various distribution scenarios [21]. The Lilliefors test statistic is calculated as follows [22]:

$$D = \max_x |F_n(z_i) - F(z_i)| \tag{18}$$

where $F_n(z_i)$ is the empirical cumulative distribution function and $F(z_i)$ is the theoretical cumulative distribution function. If the Lilliefors test statistic exceeds the critical value or if the p-value $< \alpha$, then H_0 is rejected, indicating that the data do not follow a normal distribution. The Jarque–Bera test statistic is calculated as follows [23]:

$$JB = \frac{n}{6}\left(S^2 + \frac{1}{4}(K-3)^2\right) \tag{19}$$

where n = sample size, S = sample skewness, and K = sample kurtosis. If the Jarque–Bera test statistic exceeds the critical value or if the p-value $< \alpha$, then H_0 is rejected, indicating that the data do not follow a normal distribution.

3. RESULT AND ANALYSIS

Previous studies have shown that modified EWMA-based control charts generally provide better sensitivity than conventional EWMA methods in detecting small-to-moderate process shifts [11]. Therefore, this study compares MEWMA and DMEWMA under multiple contamination scenarios using synthetic and real clinker quality data. The calibration of the control limit constant (L) and the performance evaluation for each parameter combination (λ, k) were obtained through Monte Carlo simulation and are presented in Tables 1–4.

Table 1: ARL_1 and SDRL Values of the MEWMA (Part 1)

δ	$k=-\lambda/2$			$k=-\lambda/4$			$k=\lambda/4$		
	$\lambda=0.25$	0.50	0.75	0.25	0.50	0.75	0.25	0.50	0.75
	L_M=2.8197	2.9190	2.9591	2.8437	2.9496	2.9764	2.9382	2.9887	3.0005
1.00	10.062	**11.089**	**13.813**	**9.992**	11.092	13.992	10.104	12.956	34.204
	(3.986)	(**8.597**)	(**11.393**)	(**3.801**)	(8.601)	(11.933)	(4.101)	(8.646)	(32.498)
2.00	3.774	3.478	**5.004**	3.562	**3.367**	5.142	**3.499**	3.418	5.137
	(2.342)	(1.817)	(**2.586**)	(2.245)	(**1.175**)	(2.618)	(**1.795**)	(1.288)	(2.598)
3.00	3.295	3.099	2.579	3.146	2.675	2.194	2.992	**2.183**	**1.884**
	(0.793)	(0.785)	(0.758)	(0.755)	(0.747)	(0.737)	(0.734)	(**0.693**)	(**0.633**)

Table 2: ARL_1 and SDRL Values of the MEWMA (Part 2)

δ	$k=\lambda/2$			$k=\lambda$			$k=1$		
	$\lambda=0.25$	0.50	0.75	0.25	0.50	0.75	0.25	0.50	0.75
	L_M=2.9662	3.0014	3.0037	2.9843	3.0032	2.9971	2.9986	3.0042	2.9929
1.00	10.834	16.688	65.462	11.251	44.177	193.802	104.399	160.789	254.789
	(4.211)	(13.152)	(54.258)	(4.983)	(41.298)	(181.904)	(92.691)	(137.145)	(241.758)
2.00	3.552	3.387	5.039	3.501	3.375	40.997	15.975	14.116	95.824
	(1.839)	(1.199)	(2.593)	(1.822)	(1.178)	(23.117)	(10.069)	(9.562)	(81.276)
3.00	2.664	2.398	1.894	**2.216**	2.203	17.687	2.488	6.886	45.128
	(0.723)	(0.711)	(0.688)	(**0.709**)	(0.702)	(11.752)	(0.718)	(0.852)	(35.346)

Table 3: ARL_1 and SDRL Values of the DMEWMA (Part 1)

δ	$k=-\lambda/2$			$k=-\lambda/4$			$k=\lambda/4$		
	$\lambda=0.25$	0.50	0.75	0.25	0.50	0.75	0.25	0.50	0.75
	L_{DM}=2.5635	2.7952	2.8968	2.5764	2.8186	2.9243	2.6874	2.9548	3.0055
1.00	9.650	11.055	**13.754**	9.981	**10.782**	13.950	**9.541**	12.504	34.163
	(3.489)	(8.532)	(**11.351**)	(3.651)	(**7.672**)	(11.843)	(**3.387**)	(9.283)	(32.452)
2.00	3.689	3.383	4.993	3.543	3.247	**3.658**	3.479	**3.061**	4.119
	(1.985)	(1.155)	(2.535)	(1.884)	(1.137)	(**2.198**)	(1.759)	(**1.104**)	(2.471)
3.00	3.222	3.021	2.523	3.137	2.555	2.142	2.965	2.112	**1.866**
	(0.779)	(0.743)	(0.719)	(0.741)	(0.724)	(0.711)	(0.724)	(0.691)	(**0.614**)

Table 4: ARL_1 and SDRL Values of the DMEWMA (Part 2)

δ	$k=\lambda/2$			$k=\lambda$			$k=1$		
	$\lambda=0.25$	0.50	0.75	0.25	0.50	0.75	0.25	0.50	0.75
	L_{DM}=2.7672	2.9905	3.0052	2.9070	3.0016	2.9912	3.0013	2.9952	2.9851
1.00	10.581	16.626	64.382	10.986	44.169	195.764	104.325	158.707	251.712
	(3.783)	(13.076)	(52.216)	(4.886)	(41.242)	(183.870)	(90.666)	(135.093)	(240.722)
2.00	3.451	3.184	3.763	**3.364**	3.084	40.528	16.982	14.071	95.766
	(1.738)	(1.124)	(2.255)	(**1.402**)	(1.116)	(22.227)	(10.132)	(9.407)	(80.191)
3.00	2.649	**1.891**	1.875	**2.205**	2.015	17.651	2.484	7.869	50.069
	(0.716)	(**0.643**)	(0.628)	(**0.693**)	(0.659)	(11.704)	(0.709)	(0.758)	(35.319)

Based on the simulation results of the MEWMA and DMEWMA control charts, the influence patterns of the weighting parameter λ and the additional parameter k on the ARL and SDRL values showed relatively similar tendencies for both methods. In general, λ controls the memory length of the monitoring statistic, whereas k affects the sensitivity to changes between observations. For small shifts ($\delta = 1$), increasing λ or k generally increased the ARL value, indicating that smaller λ values with longer memory structures were more effective in detecting gradual process changes. For moderate shifts ($\delta = 2$), a moderate λ value ($\lambda = 0.50$) provided more balanced performance between statistical stability and detection speed. Meanwhile, for large shifts ($\delta = 3$), increasing λ generally reduced the ARL value because greater weight on recent observations enabled faster detection of out-of-control conditions.

For several parameter combinations, particularly $k = 1$ and $k = \lambda$ when $\lambda = 0.75$, the ARL values increased substantially and did not follow the general trend. This finding indicates that excessively large λ and k values may make the monitoring statistic overly sensitive to short-term fluctuations, reducing monitoring stability. The SDRL values showed patterns generally consistent with the ARL results, where lower ARL values were typically accompanied by lower SDRL values, indicating faster and more stable detection performance. Overall, the DMEWMA control chart demonstrated better performance than MEWMA based on both ARL and SDRL measures, suggesting that the additional smoothing stage in DMEWMA improves sensitivity and stability in detecting process changes.

The optimal parameter combinations for $\delta = 3.00$ were subsequently used as the basis for implementing the control charts in the analyses of both synthetic and real data. Furthermore, the evaluation results of the MEWMA control chart using synthetic data are presented in Table 5. In addition, the evaluation results of the DMEWMA control chart using synthetic data are presented in Table 6. To improve readability, the main manuscript presents control chart plots from scenarios exhibiting the most substantial performance differences between MEWMA and DMEWMA in Figure 3 and 4. The remaining control chart plots for all evaluated λ and k combinations are provided in Appendix A.

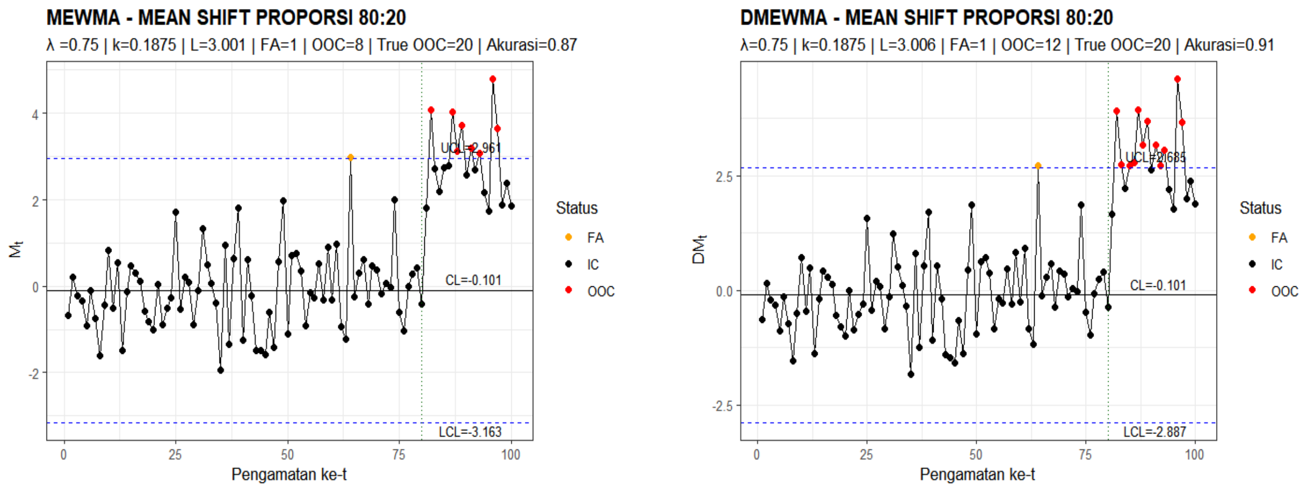
From the accuracy perspective, both methods demonstrated excellent performance, with accuracy values approaching one across most testing scenarios. Nevertheless, the improvement provided by DMEWMA over MEWMA varied depending on the contamination proportion and λ parameter values. The most pronounced improvement was observed under the 60:40 contamination scenario with $\lambda = 0.75$, where the accuracy increased from 0.82 for MEWMA to 0.90 for DMEWMA, representing an improvement of approximately 8%. Under this condition, the higher contamination proportion and larger λ value likely increased monitoring difficulty because the control charts became more sensitive to short-term fluctuations. In such situations, the additional smoothing stage in DMEWMA appears to provide more stable accumulation of process shift information, thereby improving detection performance.

Table 5: Evaluation Results of the MEWMA on Synthetic Data

Scenario	λ	k	L	False Alarm	Out of Control	True Outlier	Accuracy
60:40	0.25	0.25	2.9843	0	40	40	1.00
60:40	0.50	0.125	2.9887	0	38	40	0.98
60:40	0.75	0.1875	3.0005	0	22	40	0.82
80:20	0.25	0.25	2.9843	0	19	20	0.99
80:20	0.50	0.125	2.9887	0	18	20	0.98
80:20	0.75	0.1875	3.0005	1	8	20	0.87
90:10	0.25	0.25	2.9843	0	10	10	1.00
90:10	0.50	0.125	2.9887	0	10	10	1.00
90:10	0.75	0.1875	3.0005	0	8	10	0.98

Table 6: Evaluation Results of the DMEWMA on Synthetic Data

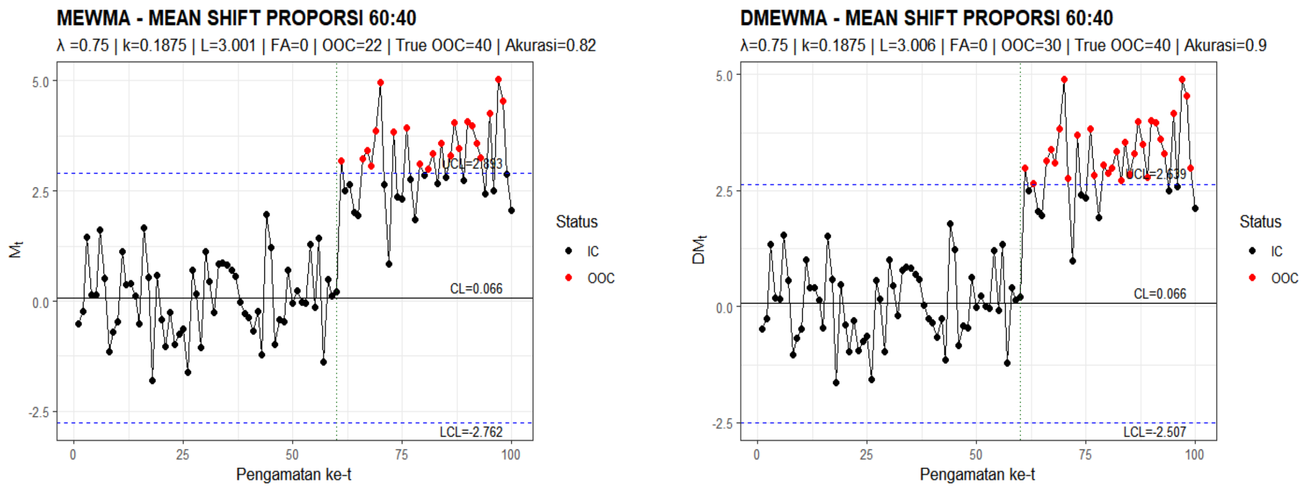
Scenario	λ	k	L	False Alarm	Out of Control	True Outlier	Accuracy
60:40	0.25	0.25	2.9070	0	39	40	0.99
60:40	0.50	0.25	2.9905	0	40	40	1.00
60:40	0.75	0.1875	3.0055	0	30	40	0.90
80:20	0.25	0.25	2.9070	0	19	20	0.99
80:20	0.50	0.25	2.9905	0	19	20	0.99
80:20	0.75	0.1875	3.0055	1	12	20	0.91
90:10	0.25	0.25	2.9070	0	10	10	1.00
90:10	0.50	0.25	2.9905	0	10	10	1.00
90:10	0.75	0.1875	3.0055	0	8	10	0.98



(a) (b)

Figure 3. Mean shift scenario 80:20 ($\lambda = 0.75$, $k = \lambda/4$) (a) MEWMA (b) DMEWMA.

In contrast, under the 90:10 scenario, the differences between MEWMA and DMEWMA were generally minimal or non-existent, with both methods producing nearly identical accuracy values across most λ settings. This result suggests that when the proportion of out-of-control observations is relatively small, the monitoring task becomes less challenging and both control charts are generally capable of maintaining stable detection performance. Consequently, the practical advantage of DMEWMA becomes more noticeable under more difficult monitoring conditions involving higher contamination intensity rather than under relatively mild contamination scenarios.

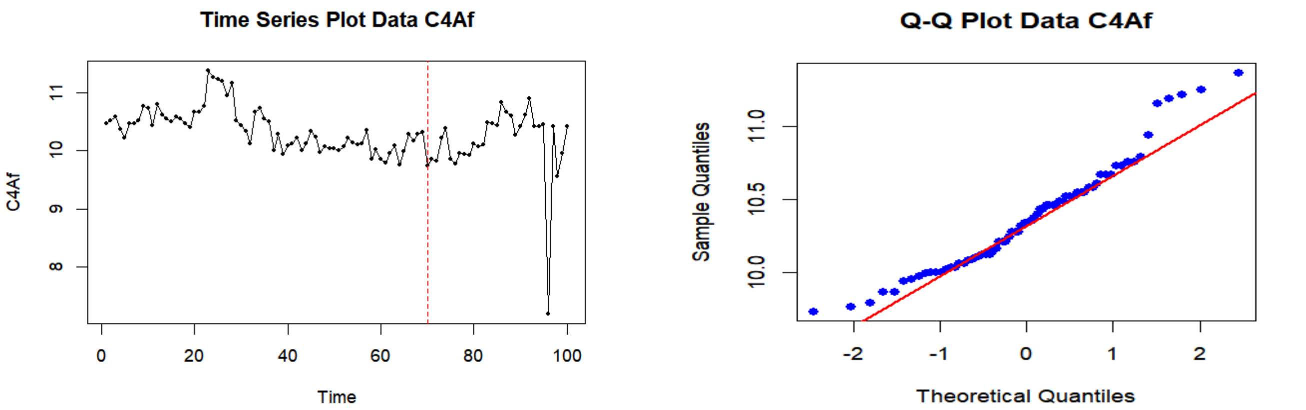


(a) **(b)**

Figure 4. Mean shift scenario 60:40 ($\lambda = 0.75$, $k = \lambda/4$) (a) MEWMA (b) DMEWMA.

Meanwhile, in terms of the weighting parameter λ, increasing the value of λ from 0.25 to 0.75 tended to reduce the number of OOC points successfully detected by both methods. However, the decline in detection capability observed in DMEWMA was generally smaller than that of MEWMA, indicating that the DMEWMA structure was relatively more stable across different weighting parameter settings. Overall, the results presented in Figures 3–4 and Table 5 and 6 indicate that DMEWMA generally provides stronger and more stable monitoring performance than MEWMA, particularly under scenarios involving higher contamination proportions and more challenging monitoring conditions.

Furthermore, the MEWMA and DMEWMA control charts were applied to secondary data in order to evaluate their performance under real-world conditions. As an initial stage, exploratory data analysis (EDA) was conducted to provide a general overview of the data characteristics and to ensure the suitability of the underlying assumptions. The results of the data visualization, normality testing, and descriptive statistics are presented in Figure 5 and Table 7.



(a) Time Series C_4AF Data **(b)** Q–Q Plot of Phase I C_4AF Data

Figure 5. Time Series and Q–Q Plot of C_4AF Data

Based on the time series plot in Figure 5, the C_4AF variable shows a relatively stable pattern throughout most observation periods, fluctuating within the range of 9.5–11.5 and indicating common cause variation without any significant upward or downward trend. However, at the 95th observation, a substantial decrease to approximately 7 was detected, indicating a potential special cause variation that may affect process stability. This extreme observation was intentionally retained in the dataset to preserve the integrity of the SPC analysis under actual operational conditions. Furthermore, based on the normality test results and the Q–Q plot, the Phase I C_4AF data can be considered approximately normally distributed, despite slight deviations in the upper tail region. This finding is supported by the Kolmogorov–Smirnov p-value of 0.6525, the Lilliefors p-value of 0.1996, and the Jarque–Bera p-value of 0.05948, all of which are greater than $\alpha = 0.05$, indicating that the normality assumption was sufficiently satisfied for implementing the MEWMA and DMEWMA control charts.

Table 7: Descriptive Statistics of Clinker Data

Parameter	Value
Mean	10.2992
Median	10.3400
Standard Deviation	0.4785
Minimum	7.1900
Q1	10.0525
Q2	10.3400
Q3	10.5275
Maximum	11.3700

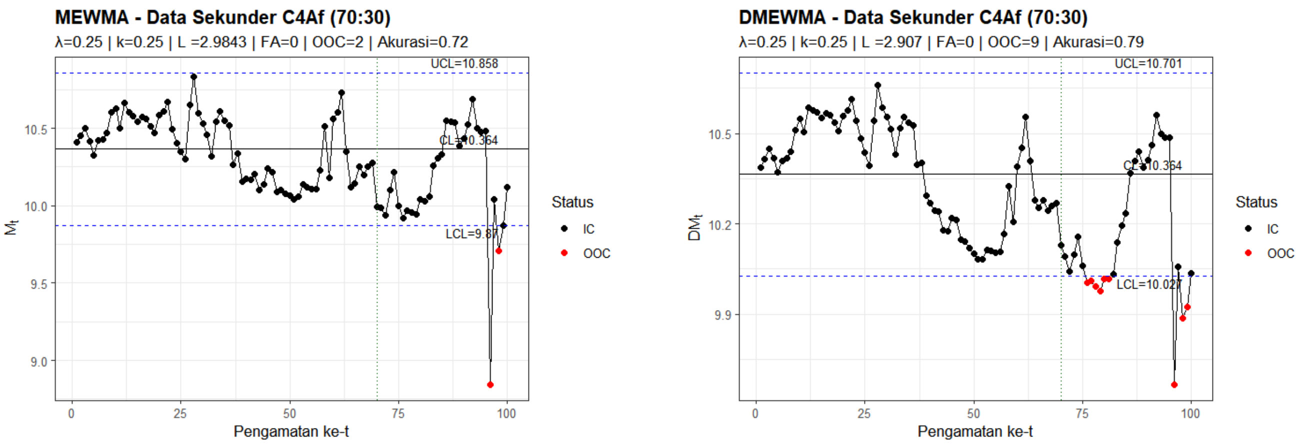
Based on Table 7, the C_4AF data tend to be symmetrically distributed with relatively low variability, as indicated by the close mean and median values and the small standard deviation. However, the wide data range and the previous time series plot suggest the presence of potential extreme values. Most observations are concentrated within the range of approximately 10.05 to 10.53, indicating a generally stable data pattern. Using the optimal parameters estimated from the Phase I data, both control charts were subsequently applied to the clinker data to detect process deviations based on false alarms, out-of-control signals, and monitoring accuracy. Furthermore, the evaluation results of the MEWMA control chart using the real clinker quality data are presented in Table 8. In addition, the evaluation results of the DMEWMA control chart using the real clinker quality data are presented in Table 9. To improve readability, the main manuscript presents control chart plots that exhibited the largest differences in monitoring performance between MEWMA and DMEWMA in Figure 6. The remaining control chart plots for all evaluated λ and k combinations are provided in Appendix B.

Table 8: Evaluation Results of the MEWMA on Clinker Data

Scenario	λ	k	L	**False Alarm**	**Out of Control**	**Accuracy**
70:30	0.25	0.25	2.9843	0	2	0.72
70:30	0.50	0.125	2.9887	0	2	0.72
70:30	0.75	0.1875	3.0005	0	1	0.71

Table 9: Evaluation Results of the DMEWMA on Clinker Data

Scenario	λ	k	L	**False Alarm**	**Out of Control**	**Accuracy**
70:30	0.25	0.25	2.9070	0	9	0.79
70:30	0.50	0.25	2.9905	0	2	0.72
70:30	0.75	0.1875	3.0055	0	1	0.71



(a) **(b)**

Figure 6. Clinker data ($\lambda = 0.25$, $k = \lambda$) (a) MEWMA (b) DMEWMA.

Based on the evaluation results presented in Tables 8 and 9 both the MEWMA and DMEWMA control charts demonstrated good stability under in-control conditions, as indicated by the absence of false alarms across all λ values. In terms of detection performance, the DMEWMA control chart produced substantially more out-of-control

(OOC) signals than the MEWMA control chart at $\lambda = 0.25$, where DMEWMA detected 9 OOC observations compared with only 2 observations detected by MEWMA. This result was also followed by an improvement in monitoring accuracy from 0.72 for MEWMA to 0.79 for DMEWMA. The stronger improvement observed at $\lambda = 0.25$ may be associated with the longer memory structure produced by smaller λ values. Under smaller smoothing parameters, the monitoring statistics place greater emphasis on historical observations, allowing gradual process deviations in the real clinker data to accumulate more consistently over time. Because DMEWMA applies an additional smoothing stage, the memory effect becomes more pronounced at lower λ values, thereby improving sensitivity toward subtle and persistent process changes under actual industrial conditions.

In contrast, at $\lambda = 0.50$ and $\lambda = 0.75$, both control charts produced identical numbers of OOC detections and similar accuracy values. Under larger λ values, both MEWMA and DMEWMA become more responsive to recent observations and less dependent on historical accumulation. Consequently, the additional smoothing mechanism in DMEWMA provides less incremental improvement, causing the monitoring performances of both methods to become relatively similar. Overall, these findings indicate that the practical advantage of DMEWMA on the real clinker dataset becomes more pronounced under smaller λ values, where the stronger memory structure is more effective in capturing gradual process deviations commonly observed in actual industrial monitoring data.

From a practical perspective, the superior monitoring capability of DMEWMA indicates that potential process deviations in clinker quality may be identified earlier and more consistently during industrial monitoring activities. Earlier detection may support timely corrective actions, improve production consistency, reduce material waste, and strengthen quality assurance. In addition, improved monitoring sensitivity may also help identify unstable operating conditions related to combustion irregularities, excessive dust generation, and process imbalance, thereby supporting occupational health, industrial safety, and environmental quality control in cement manufacturing. Although this study focused on clinker quality monitoring through the C_4AF variable, the proposed monitoring framework may also be applicable to other industrial environments such as pharmaceutical manufacturing, food production, chemical processing, and water treatment systems. The findings are consistent with previous studies [10, 11], which reported that modified EWMA-based control charts provide better sensitivity than conventional EWMA methods. The stronger performance of DMEWMA is likely associated with its additional smoothing stage, which improves sensitivity toward gradual and persistent process shifts while maintaining monitoring stability.

It should be noted that the synthetic data evaluation presented in Tables 5-9 was conducted using a single synthetic data realization for each contamination scenario, while the real-data evaluation was based on a single clinker dataset consisting of 30 Phase II observations. The primary purpose of these analyses was to illustrate the practical monitoring behavior of the MEWMA and DMEWMA control charts in terms of false alarm occurrence, out-of-control detection capability, monitoring accuracy, and first signal detection under representative monitoring conditions. In contrast, the statistical performance of the control charts was primarily evaluated through Monte Carlo simulation with 20,000 replications using ARL and SDRL measures. Therefore, the synthetic and real-data analyses should be viewed as complementary case studies that demonstrate the practical application of the control charts under the scenarios considered in this study. Further evaluation using multiple synthetic data realizations and larger real datasets may provide additional evidence regarding the consistency of the observed performance differences.

4. CONCLUSION

Based on the comparative performance analysis of the MEWMA and DMEWMA control charts using both synthetic and real clinker quality data, the following conclusions were obtained:

a. The Monte Carlo simulation successfully generated optimal parameter combinations for the MEWMA and DMEWMA control charts under the target condition of $ARL_0 \cong 370$. Based on the ARL and SDRL measures, DMEWMA generally outperformed MEWMA in detecting process mean shifts.
b. For the same value of k, smaller λ values generally provided better sensitivity for detecting gradual and small process shifts due to the stronger historical memory structure of the monitoring statistics, whereas larger λ values tended to respond more rapidly to large process shifts. However, excessively large combinations of λ and k reduced monitoring stability because short-term fluctuations became amplified within the monitoring statistic.
c. Under synthetic contamination scenarios, DMEWMA generally demonstrated stronger and more stable monitoring performance than MEWMA, particularly under higher contamination conditions. The additional smoothing stage in DMEWMA allowed persistent process deviations to accumulate more consistently while reducing the influence of short-term random fluctuations, resulting in improved detection capability and more stable monitoring accuracy.
d. The application to real clinker quality data also showed that DMEWMA provided more effective detection capability under actual industrial monitoring conditions, especially at lower λ values where the monitoring structure retained stronger historical memory. These findings indicate that DMEWMA is particularly

suitable for monitoring environments involving gradual and persistent process deviations.

e. From a practical industrial perspective, improved monitoring sensitivity in clinker quality monitoring may support earlier identification of unstable operating conditions associated with combustion irregularities, excessive dust generation, and process imbalance in cement manufacturing systems. Consequently, more effective process monitoring may contribute not only to improved product quality consistency and operational reliability but also to occupational safety management, environmental quality control, and industrial health protection within cement manufacturing environments.

4.1 Limitations and Future Directions

Based on the limitations and findings of this study, several suggestions for future research are proposed as follows:

a. Future studies are recommended to evaluate the performance of MEWMA and DMEWMA under more complex non-normal distributions and different contamination patterns in order to investigate the robustness of the proposed methods more comprehensively.
b. The application of the proposed methods should be extended to larger and more diverse industrial datasets to improve the generalization of the monitoring results across different industrial process environments.
c. Future research may develop multivariate statistical process control (MSPC) approaches based on MEWMA and DMEWMA for monitoring multiple correlated quality characteristics simultaneously. Such multivariate extensions may provide broader practical benefits in industrial quality management environments where multiple process variables jointly influence product quality, operational reliability, and process safety.
d. Further studies may also integrate robust estimation techniques to improve monitoring performance under complex industrial process conditions and heterogeneous process variability patterns.

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