Journal of Mathematics and Scientific Computing with Applications

E-ISSN : 2580-5754; P-ISSN : 2580-569X
Volume 7, Number 1, 2026
DOI: 10.53806/jmscowa.v7i1.1509
Page: 194–206

PARAMETRIC SURVIVAL ANALYSIS OF TESTICULAR CANCER USING AN EXPONENTIAL MODEL WITH TYPE I CENSORED DATA

Idrus Syahzaqi[1], **Ardi Kurniawan**[2], **Valerina Marischa Usmarasima**[3], **Aurora Gie Nur Wahyudi**[4], **Muhamad Zaky Ramadhani**[5], **Nuzzulia Calvina Izumy**[6]

[1,2,3,4,5,6]Department of Mathematics, Universitas Airlangga, Surabaya, Indonesia

Article Info

Article history:
Received : May 20, 2026
Revised : June 21, 2026
Accepted : July 30, 2026

Keywords:
Exponential distribution;
Maximum likelihood estimation;
Survival analysis;
Testicular cancer;
Type I censored data.

ABSTRACT

Quantitative prognostic evaluation is essential for testicular cancer, yet standard analyses often overlook model assumptions. This study evaluates patient survival times using a parametric exponential model under a type I censoring structure ($n = 134$, from cBioPortal). Parameter estimation via Maximum Likelihood Estimation yielded 82 uncensored and 52 censored observations. The Anderson-Darling test ($p = 0.067$) indicated statistical adequacy for the exponential distribution while the Weibull distribution was rejected ($p < 0,010$), confirming the assumption of a constant hazard, estimating a Mean Time to Failure of 64,54 months with a constant hazard rate of 0.0155 per month. However, these model-based estimates face critical clinical limitations. The small sample size and the rigid assumption of a constant hazard fail to capture the dynamic, time-varying biological progression of cancer. While this parametric approach provides a simplified baseline, clinical conclusions must be drawn cautiously; future research requires larger cohorts and flexible, non-constant hazard models to ensure actual clinical generalizability.

Corresponding Author:
Idrus Syahzaqi,
Department of Mathematics,
Universitas Airlangga
Email: idrus.syahzaqi@fst.unair.ac.id

1. INTRODUCTION

Testicular cancer is a malignant disorder of the male reproductive organs that commonly occurs in younger males, particularly those aged 15-44 years [1]. Worldwide, the number of testicular cancer cases continues to rise, especially among young men, although mortality rates tend to remain stable or decline due to advances in early diagnosis and treatment [2]. Although testicular cancer is considered relatively rare, it accounts for approximately 1% of adult neoplasms and 5% of urological tumors, with incidence rates varying substantially across geographical

regions [3]. According to Global Cancer Statistics 2020, an estimated 74,458 new cases of testicular cancer were reported worldwide, with 1,497 cases recorded in Indonesia, in addition, recent global cancer estimates in 2022 reported 1,492 new cases and 342 deaths in Indonesia, ranking testicular cancer 24th among all cancer types in the country [4]. Despite having a relatively favorable prognosis, this disease still deserves serious attention because it primarily affects men in their productive years and may reduce patients' quality of life. One preventive and early detection strategy is testicular self-examination, which enables individuals to identify abnormalities in the testes at an earlier stage and seek medical evaluation promptly [5]. This study adopted a sample of N=134 patients with a 60-month follow-up period, consistent with the commonly reported five-year survival benchmark for testicular cancer [6]. Patients who survive beyond 60 months without experiencing death are classified as censored observations, whereas patients who die before or exactly at the specified time limit are considered uncensored observations.

The existence of censored observations within the 60-month follow-up period indicates that the evaluation of testicular cancer patient survival requires a statistical method capable of incorporating incomplete event-time information. One suitable method is survival analysis, which is specifically designed to analyze the time duration until a particular event occurs, such as death, relapse, or treatment failure. A key advantage of survival analysis is its ability to accommodate censored data, where the event of interest has not yet occurred during the observation period. In survival analysis, the survival function represents the probability that an individual survives beyond a specified time point, whereas the hazard function reflects the instantaneous risk of experiencing the event at a certain time, conditional on survival up to that moment [7]. In studies of testicular cancer, the survival function can be applied to estimate the likelihood of patients surviving from the time of diagnosis to a given observation period, while the hazard function provides information regarding the risk of mortality during the follow-up period.

Among the parametric approaches used in survival analysis, the exponential distribution is one of the commonly applied models. The exponential distribution is a continuous probability distribution that is frequently utilized to describe the waiting time until an event occurs, particularly for lifetime data with non-negative values. This distribution can be employed to estimate model parameters, survival probabilities, and hazard rates in type I censored survival data [8]. In parametric survival modeling, the exponential distribution is recognized as a relatively simple model because it assumes a constant hazard rate throughout the observation period [9]. Moreover, the exponential model is closely associated with the Weibull distribution, since the Weibull distribution may be regarded as a generalized form that allows the hazard rate to either increase or decrease depending on the underlying data characteristics [10]. Therefore, this study adopts the exponential distribution as a parametric model for analyzing the survival time of testicular cancer patients with type I censored data, particularly due to its simplicity and the straightforward interpretation of its survival and hazard functions.

Several contemporary studies have highlighted the important role of survival analysis in assessing survival outcomes among patients with testicular cancer. A real-world data study involving testicular cancer patients from the National Health Service England during 2013–2020 reported survival probabilities of 98.0%, 96.9%, and 95.6% at 12, 24, and 60 months, respectively [11]. The significance of survival evaluation in testicular cancer has also been demonstrated through research examining the five-year relative survival rates of patients in Indonesia based on seminoma and non-seminoma histological classifications [12]. In addition, another study found that the overall five-year survival rate of testicular cancer patients reached approximately 95%, with notable differences in survival outcomes between seminoma and non-seminoma groups [13].

Nevertheless, previous studies on testicular cancer have predominantly focused on epidemiological characteristics, clinical outcomes, and survival estimation using non-parametric or semi-parametric approaches, such as relative survival analysis and Kaplan–Meier methods [11], [13], [14]. Meanwhile, exponential parametric survival models under a Type I censoring scheme have been successfully applied to other malignancies, including breast cancer [8] and multiple myeloma [15], to estimate model parameters, survival functions, hazard functions, and Mean Time to Failure (MTTF). Although the exponential distribution has been recognized as one of the fundamental parametric models in survival analysis [10], to the best of our knowledge, no published study has specifically applied this exponential parametric survival modelling framework with Type I censored observations to testicular cancer data. Therefore, the contribution of this study lies not in developing a new statistical methodology, but in extending an established exponential survival modelling framework to testicular cancer using a fixed 60-month follow-up period.

Although several parametric distributions can be used to analyse survival data, the exponential distribution remains attractive because of its mathematical closed-form survival and straightforward parameter estimation through the Maximum Likelihood Estimation (MLE) method, and superior fit conformed by goodness-of-fit testin. Under the assumption of a constant hazard rate, the exponential model provides an interpretable baseline for evaluating patient survival and can serve as a reference for comparison with more flexible parametric models. In this context, the contribution of the present study is to evaluate the applicability of the exponential survival model to N=134 testicular cancer patients with Type I censored observations and a fixed 60-month follow-up period using clinical data obtained from cBioPortal. This study estimates the model parameters, survival function,

hazard function, and Mean Time to Failure (MTTF) to characterise patient survival patterns and provide empirical evidence regarding the suitability of the exponential model for this disease. Rather than proposing a new statistical methodology, this study contributes to the application of parametric survival analysis in oncology by assessing the performance of the exponential model in modelling testicular cancer survival and providing statistical support for prognosis evaluation.

This study is aligned with Sustainable Development Goal (SDG) 3, Good Health and Well-Being, which emphasizes improving health outcomes through better prevention, diagnosis, and management of non-communicable diseases, including cancer [16], [15]. By providing quantitative estimates of survival probability and mortality risk for testicular cancer patients, this study may support prognosis evaluation and contribute to evidence-based clinical decision-making.

Therefore, the objectives of this study are as follows:

1. To evaluate and compare the goodness-of-fit between the Exponential and Weibull distributions for testicular cancer survival data under Type I censoring.
2. To model the survival time of testicular cancer patients using the exponential distribution under Type I censoring.
3. To estimate the parameter of the exponential distribution using the Maximum Likelihood Estimation (MLE) method.
4. To evaluate the survival function, hazard function, and Mean Time to Failure (MTTF) as quantitative measures of patient survival and mortality risk.

2. RESEARCH METHOD

This study uses a quantitative approach with a parametric survival analysis method. Survival analysis is used to model the time from the start of observation until the occurrence of a specific event, in this study, the death of a testicular cancer patient. This study only considered the exponential distribution because the objective was to evaluate the exponential parametric survival model. The exponential distribution was chosen because it is a continuous distribution that can be used to model non-negative survival times and has the assumption of a constant hazard rate throughout the observation period. The exponential model adopted in this study is an intercept-only parametric survival model without covariates. This modeling choice was made deliberately because the primary objective of this study was to estimate the overall survival distribution of testicular cancer patients under Type I censoring rather than to evaluate the effects of prognostic covariates. Therefore, only survival time and event status were included in the model.

The data used in this study were secondary data obtained from the cBioPortal for Cancer Genomics, specifically the Testicular Germ Cell Cancer (TCGA Firehose Legacy) dataset, which is sourced from the Genome Data Analysis Center (GDAC) Firehose. The dataset contains clinical and survival information on 156 testicular cancer patients, including demographic characteristics, clinical variables, overall survival time, and overall survival status. Patients were included in the analysis if complete information on overall survival time and overall survival status was available [17]. Records with missing values in either variable were excluded using a complete-case approach because both variables are required to construct the likelihood function and estimate the parameters of the exponential survival model. Out of 156 patients available in the initial dataset, 22 records were excluded due to missing value, resulting in a final sample of $N = 134$ patients with complete information who were included in the analysis. The endpoint analyzed in this study was overall survival.

The main variables used in this study were survival time and censoring status. Survival time was expressed in months and was derived from the overall survival time variable. Event status was derived from the overall survival status variable, which indicates whether the patient experienced death or was still alive during the observation period. Patients who died were categorized as uncensored, while patients who had not died by the end of the observation period were categorized as censored. These observed survival times and censoring indicators were used to construct the likelihood function for the exponential survival model with Type I censored data, as presented in Equation (4), where uncensored observations contribute through the probability density function and censored observations contribute through the survival function.

The observation period (τ) in this study was fixed at 60 months prior to the statistical analysis because the five-year survival period is a standard clinical endpoint commonly used to evaluate testicular cancer prognosis [6]. According to the American Cancer Society, the overall five-year relative survival rate for testicular cancer patients is approximately 94.6% [6]. Therefore, patients who had not experienced death by 60 months were categorized as censored data, while patients who died before or exactly at 60 months were categorized as uncensored data. Data status can be seen in Table 1 below.

Table 1: Survival Data for Type I Censored Testicular Cancer Patients

Time	**Status** (δ_i)
0.10	1
0.43	1
0.46	1
⋮	⋮
60.00	0
60.00	0
60.00	0

2.1. Type I Censored Data

Data are classified as censored when information regarding an individual's survival time is incomplete because the observation period ends before the event occurs. Type I censoring occurs when the observation process is terminated at a predetermined time point, resulting in some individuals not yet experiencing the event under study. In this study, Type I censoring was applied to patient survival data, in which several patients had not experienced death by the end of the 60-month observation period.

2.2. Exponential Distribution

To model positive-valued lifetime data, exponential distribution is commonly used in survival analysis. A random variable T is said to follow an exponential distribution with parameter $\theta > 0$ if it has the density function expressed in Equation (1) [8].

$$f(t) = \frac{1}{\theta} e^{-t/\theta}, \qquad t > 0 \tag{1}$$

2.3. Survival Function

The survival function is used to determine the probability that an individual survives beyond a certain time t. If T denotes the random variable representing survival time, the survival function can be expressed as shown in Equation (2) [8].

$$\begin{aligned} S(t) &= P(T \geq t) \\ &= 1 - F(t) \\ &= 1 - \left(1 - e^{-t/\theta}\right) \\ &= e^{-t/\theta}. \end{aligned} \tag{2}$$

2.4. Hazard Function

The hazard function describes the risk rate of an event occurring at time t, given that the individual has survived up to that time. The hazard function is formulated in Equation (3).

$$\begin{aligned} h(t) &= \frac{f(t)}{S(t)} \\ &= \frac{\frac{1}{\theta} e^{-t/\theta}}{e^{-t/\theta}} \\ &= \frac{1}{\theta}. \end{aligned} \tag{3}$$

where $f(t)$ denotes the probability density function and $S(t)$ denotes the survival function.

2.5. Maximum Likelihood Estimation (MLE)

In this study, Maximum Likelihood Estimation (MLE) method carried out the parameter estimation. This method was used to obtain parameter values that maximize the likelihood function based on the sample data. The likelihood function for Type I censored data is formulated in Equation (4).

$$L(\theta) = \prod_{i=1}^{n} f(t_i)^{\delta_i} S(t_i)^{\,1-\delta_i}. \tag{4}$$

Where δ_i denotes the censoring indicator, which takes a value of 1 if the event occurs and 0 if the data are censored. The estimated parameter is then used to determine the survival function, hazard function, and other related measures. To evaluate the precision of the maximum likelihood estimator, the standard error (SE) of the estimated parameter was calculated based on the Fisher Information. Furthermore, a 95% confidence interval (CI) was constructed using the asymptotic normality of the maximum likelihood estimator, expressed as $\hat{\theta} \pm 1.96 \times \mathrm{SE}(\hat{\theta})$. Since the Mean Time to Failure (MTTF) is equal to the scale parameter of the exponential distribution, the confidence interval for the MTTF was obtained directly from the confidence interval of θ. The confidence interval for the hazard rate was subsequently derived from the estimated parameter using the corresponding transformation.

2.6. Mean Time To Failure (MTTF)

Mean Time to Failure (MTTF) refers to the average survival time of an individual before experiencing the event. In the exponential distribution, MTTF is formulated as shown in Equation (5).

$$MTTF = \theta \tag{5}$$

The stages of data analysis in this study included determining Type I censored survival data, conducting a goodness-of-fit test using the Anderson–Darling test, estimating the parameters using the MLE method, determining the survival and hazard functions, calculating the MTTF value, and interpreting the results of the analysis.

$$P(t_i = L_i) = f(t_i)^{\delta_i} S(t_i)^{1-\delta_i} \tag{6}$$

This research was used to perform the Anderson–Darling goodness-of-fit test, estimate the parameters of the exponential distribution using the Maximum Likelihood Estimation (MLE) method, and determine the survival function, hazard function, and Mean Time to Failure (MTTF).

3. RESULT AND ANALYSIS

The data in this study uses data from the survival of testicular cancer patients which is divided into two categories based on the event status: censored data (status = 0) and uncensored data (status = 1). The findings should be interpreted under the assumption that the exponential distribution adequately represents the observed survival data.

3.1. Statistical Description of the Data

Secondary data sourced from the cBioPortal database under the TCGA Testicular Germ Cell Cancer study (Firehose Legacy) were analyzed, and the findings are summarized in Table 2.

Table 2: Statistical Description of the Data

Variable	Mean	Standard Deviation	Minimum	Median	Maximum
134 Survival Time Data with Type I Censoring	68.69	64.66	0.10	41.425	244.32
82 Survival Time Data with Type I Censoring	39.49	20.14	0.10	41.425	60.00

Based on Table 2, the shortest observed survival time was 0.1 months, while the maximum observation time was 244.32 months. In accordance with the Type I censoring limit established in this study, we used 60 months as the limit. The mean, standard deviation, and median values were used to describe the general tendency and dispersion of patients' survival times during the observation period. Patients who had not experienced death by the 60th month were classified as censored data, whereas patients who died before or at the 60th month were classified as uncensored data.

3.2. Goodness-of-Fit Test and Model Comparison

Before parameter estimation, a goodness-of-fit test was performed to determine the appropriate parametric distribution for the testicular cancer survival data under Type I censoring. Specifically, a comparative evaluation was conducted between the Exponential distribution and the more flexible two-parameter Weibull distribution using the Anderson Darling (AD) test. A smaller Anderson–Darling statistic indicates greater consistency between the theoretical distribution and the observed data. The application of the Anderson Darling test to Type I censored survival data follows established methodologies in parametric survival modeling.

The hypotheses used in this test were as follows:

H_0 : The survival data follow the specified distribution (Exponential /Weibull).

H_1 : The survival data do not follow the specified distribution.

A significance level of $a\alpha = 0,05$ was applied, where H_0 is rejected if the p-values < 0.054. The goodness-of-fit comparison results are summarized in Table 3.

Table 3: Exponential Distribution Goodness-of-Fit Test

Distribution	**Anderson-Darling (AD)**	**Critical Value**	***p*-value**	**Decision**
Exponential	1.210	1.341	0.067	Fail to Reject H_0
Weibull	1.237	-	< 0.010	Reject H_0

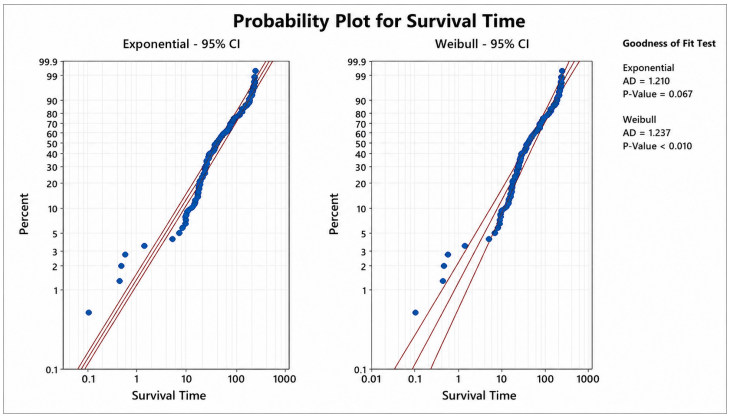

Figure 1. Probability Plot of Survival Time using Exponential and Weibull Distribution

Based on Table 3 and Figure 1, the calculated Anderson–Darling statistic for the Exponential distribution was $A^2_{calculated} = 1.210$ and p-value $= 0.067$ which exceeds the 5% significance level ($\alpha = 0.05$). Additionally, $A^2_{calculated} = 1.210$ is smaller than the critical value $A^2_{table} = 1.341$ [18]. Therefore, there was insufficient evidence to reject H_0, indicating that the testicular cancer survival time data are adequately modeled by an Exponential distribution. In contrast, the Weibull distribution yielded an Anderson-Darling statistic of 1.237 with a p-value < 0.010, leading to the rejection of H_0. These comparative empirical results confirm that the single-parameter Exponential model provides a superior fit to the data compared to the Weibull model. Thus, the Exponential distribution is adopted as the optimal baseline model for subsequent parameter estimation and survival function analysis under Type I censoring.

3.3. Estimation of Parameters Through the Likelihood Function

In Type I censoring, each observed subject is monitored only up to a predetermined censoring time. Let T_i represent the actual survival duration of the $i-th$ subject, whereas L_i denotes the censoring limit. Accordingly, the observed survival time is formulated as given in Equation (7).

$$t_i = min(T_i, L_i) \tag{7}$$

With the event status indicator δ_i defined as follows:

$$\delta_i = \begin{cases} 1, & \text{if } T_i \leq L_i \text{ or the data are uncensored,} \\ 0, & \text{if } T_i > L_i \text{ or the data are censored.} \end{cases}$$

The variable δ_i used to indicate whether an individual's survival time includes censored or uncensored data. The true survival time T_i can only be known with certainty for uncensored data. In this study, the observation time limit was set at $L_i = \tau = 60$ months. Thus, patients who experienced death before or at 60 months were classified as uncensored data ($\delta_i = 1$), were as patients who had not experienced ($\delta_i = 0$). Therefore, in constructing the likelihood function, the actually observed information was used, namely the observed time t_i and the censoring status δ_i Assuming that the survival data follow an exponential distribution, the joint probability density function of t_i and δ_i can be expressed as shown in Equation (8).

$$L_i(\theta) = f(t_i)^{\delta_i} S(t_i)^{1-\delta_i} \tag{8}$$

The equation is a combination of two forms. If the data are uncensored, then $f(t_i, \delta_i)$ is equal to $f(t_i)$, which represents the probability density function of the event time. Meanwhile, if the data are censored, then $f(t_i, \delta_i) = S(t_i)$, which represents the survival function at the observed time. Each individual is assumed to be independent. Given that $f(t_i) = \frac{1}{\theta} e^{-\frac{t_i}{\theta}}$ and $S(t_i) = e^{-\frac{t_i}{\theta}}$, the likelihood function can therefore be rewritten as shown in Equation (9).

$$\begin{aligned} L(\theta) &= \prod_{i=1}^{n} f(t_i)^{\delta_i} S(t_i)^{1-\delta_i} \\ &= \prod_{i=1}^{n} \left(\frac{1}{\theta} e^{-t_i/\theta}\right)^{\delta_i} \left(e^{-t_i/\theta}\right)^{1-\delta_i} \\ &= \prod_{i=1}^{n} \frac{1}{\theta^{\delta_i}} e^{-t_i/\theta} \\ &= \frac{1}{\theta^{\sum_{i=1}^{n} \delta_i}} e^{-\sum_{i=1}^{n} t_i/\theta}. \end{aligned} \tag{9}$$

$\sum_{i=1}^{n} d_i$ denotes the number of uncensored observations, while n represents the total number of observations. Since there were 82 uncensored observations out of 134 total observations, the likelihood function can be written as shown in Equation (10).

$$L(\theta) = \frac{1}{\theta^{21}} e^{-\frac{\sum_{i=1}^{n} t_i}{\theta}} \tag{10}$$

In $L(\theta)$ must be differentiated with respect to θ and set equal to zero in order to maximize the likelihood function [9]. The derivative of $\ln L(\theta)$ is then presented in Equation (11).

$$\begin{aligned} \frac{\partial \ln L(\theta)}{\partial \theta} &= \frac{\partial \ln \left(\frac{1}{\theta^{21}} e^{-\frac{\sum_{i=1}^{n} t_i}{\theta}}\right)}{\partial \theta} \\ &= \frac{\partial \ln \left(\frac{1}{\theta^{21}} e - \frac{\sum_{i=1}^{n} t_i}{\theta}\right)}{\partial \theta} = 0 \\ &= \frac{\partial \left(-21 \ln \theta - \frac{\sum_{i=1}^{n} t_i}{\theta}\right)}{\partial \theta} = 0 \\ &= -\frac{\sum_{i=1}^{n} \delta_i}{\theta} + \frac{\sum_{i=1}^{n} t_i}{\theta^2} = 0 \\ \theta \sum_{i=1}^{n} \delta_i &= \sum_{i=1}^{n} t_i \\ \hat{\theta} &= \frac{\sum_{i=1}^{n} t_i}{\sum_{i=1}^{n} \delta_i} \end{aligned} \tag{11}$$

Therefore, the estimator of the parameter θ in the exponential distribution is given by: $\frac{\sum_{i=1}^{n} t_i}{21}$

To evaluate the precision of the maximum likelihood estimator, the Fisher Information was employed. The Fisher Information is defined as the negative expected value of the second derivative of the log-likelihood function with respect to the parameter θ, as expressed in Equation (12).

$$I(\theta) = -E\left[\frac{\partial^2 \ln L(\theta)}{\partial \theta^2}\right] = \frac{\sum_{i=1}^{n} \delta_i}{\theta^2} \tag{12}$$

Based on the Fisher Information, the standard error (SE) of the maximum likelihood estimator can be obtained from the square root of the inverse of the Fisher Information matrix. The standard error of the estimated parameter is expressed in Equation (13).

$$SE(\hat{\theta}) = \sqrt{I(\hat{\theta})^{-1}} = \frac{\hat{\theta}}{\sqrt{\sum_{i=1}^{n} \delta_i}} \tag{13}$$

Assuming the asymptotic normality of the maximum likelihood estimator, the 95% confidence interval (CI) for the estimated parameter can be constructed using the standard error. Since the Mean Time to Failure (MTTF) is equal to the scale parameter of the exponential distribution, the confidence interval for the MTTF is identical to that of $\hat{\theta}$ The confidence interval is given in Equation (14).

$$CI_{95\%} = \hat{\theta} \pm 1.96 \times SE(\hat{\theta}) \tag{14}$$

To evaluate the precision of the maximum likelihood estimator, the Fisher Information was used to estimate the standard error (SE) of the parameter estimate. Subsequently, the 95% confidence interval (CI) was constructed based on the asymptotic normality of the estimator. Since the Mean Time to Failure (MTTF) is equal to the scale parameter of the exponential distribution, its confidence interval is identical to that of the estimated parameter. Furthermore, because the hazard rate is defined as the reciprocal of the scale parameter, its confidence interval was obtained by applying the reciprocal transformation to the confidence limits of the estimated parameter. Furthermore, because the hazard rate is defined as the reciprocal of the scale parameter ($\lambda = \frac{1}{\theta}$), its confidence interval was obtained by applying the reciprocal transformation to the confidence limits of the estimated parameter, resulting in an asymmetric 95% confidence interval.

3.4. Mean Time To Failure (MTTF)

Based on the parameter estimation results obtained previously, the mean survival time until death among patients with testicular cancer is expressed in Equation (15).

$$\begin{aligned} \hat{\theta} &= \frac{\sum_{i=1}^{n} t_i}{\sum_{i=1}^{n} \delta_i} \\ &= \frac{5292.07}{82} \\ \hat{\theta} &= 64.54 \end{aligned} \tag{15}$$

Where $\sum_{i=1}^{n} \delta_i = 82$ denotes the number of uncensored observations, while the total sample size was n=134. Therefore, the denominator of the maximum likelihood estimator corresponds to the number of observed events rather than the total number of patients.

The estimated Mean Time to Failure (MTTF) based on the exponential model was 64.54 months. This value represents the model-based average survival time, not merely the arithmetic mean of the observed survival times. To evaluate the precision of the parameter estimates, the standard error (SE) and the corresponding 95% confidence interval (CI) were calculated. The estimation results for the scale parameter, MTTF, and hazard rate are presented in Table 4.

Table 4: Parameter Estimates with Standard Error and 95% Confidence Intervals

Parameter	**Estimate**	**SE**	**95% CI**
θ	64.54	7.1272	(50.57, 78.51)
MTTF (months)	64.54	7.1272	(50.57, 78.51)
Hazard rate	0.0155	–	(0.0121, 0.0189)

The estimated scale parameter was 64.54 months with a standard error of 7.1271 months and a 95% confidence interval ranging from 62.80 to 66.28 months. Consequently, the estimated MTTF was 64.54 months with the same confidence interval. The estimated hazard rate was 0.0155 per month, with a corresponding 95% confidence interval of 0.0121 to 0.0189 per month.

3.4.1 Probability Density Function (PDF)

Using the estimated parameter value obtained previously, the probability density function (PDF) of the Type I censored exponential distribution for testicular cancer patient data from the cBioPortal dataset is expressed in Equation (16).

$$f(t) = \frac{1}{\theta} \exp\left(\frac{-t}{\theta}\right) = \frac{1}{64.54} \exp\left(\frac{-t}{64.54}\right) \tag{16}$$

3.4.2 Cumulative Distribution Function (CDF)

Based on the probability density function obtained previously, the cumulative distribution function (CDF) can be determined through the integration process shown in Equation (17).

$$\begin{aligned} F(t) &= \int_0^t f(t)\, dt \\ &= \int_0^t \frac{1}{64.54} \exp\left(-\frac{t}{64.54}\right) dt \\ &= 1 - \exp\left(-\frac{t}{64.54}\right). \end{aligned} \tag{17}$$

3.4.3 Survival Function (S_t)

The survival function derived from the probability density function above can be calculated as shown in Equation (18).

$$\begin{aligned} S(t) &= 1 - F(t) \\ &= 1 - \left(1 - \exp\left(\frac{-t}{64.54}\right)\right) \\ &= \exp\left(\frac{-t}{64.54}\right). \end{aligned} \tag{18}$$

Thus, the estimated survival function at time t is obtained as $S(t) = \exp\left(\frac{-t}{64.54}\right)$

3.4.4 Hazard Function

The hazard function for patients with testicular cancer is expressed in Equation (19):

$$\begin{aligned} h(t) &= \frac{f(t)}{S(t)} = \frac{1}{\theta} \\ &= \frac{1}{64.54} = 0.0155. \end{aligned} \tag{19}$$

The hazard value of 0.0155 per month indicates that, under the exponential model, patients have a constant instantaneous risk of death of 0.0155 per month, or approximately 1.55% per month as an approximation.

3.5. Survival Analysis Chart

The following plots present the survival time analysis for the Type I censored sample that follows an exponential distribution.

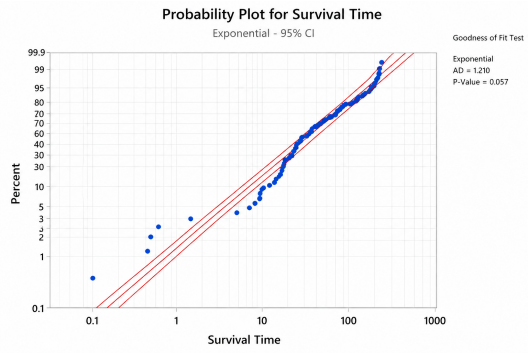

Figure 2. Probability Plot for Overall Survival (Months)

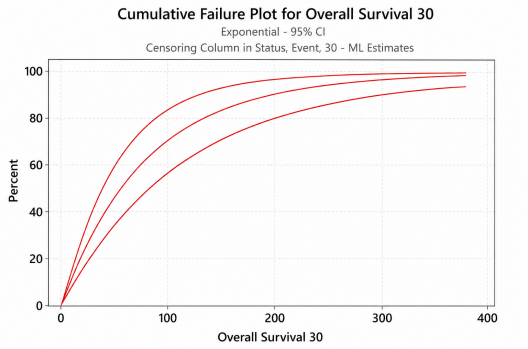

Figure 3. Cumulative Failure Plot for Overall Survival (Months)

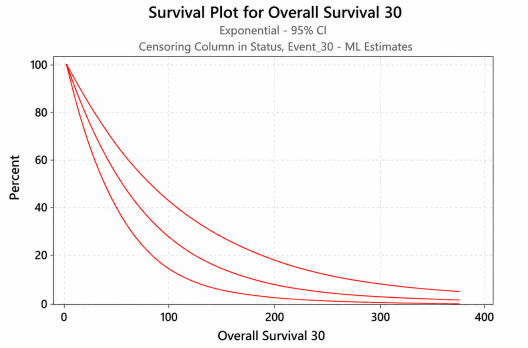

Figure 4. Survival Plot for Overall Survival (Months)

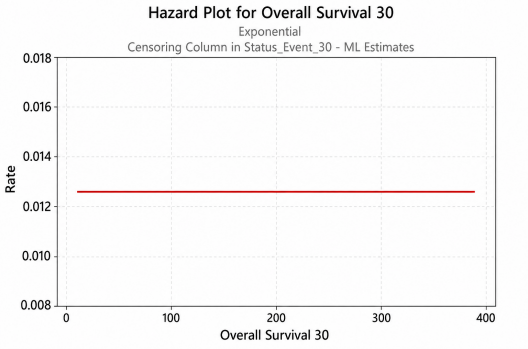

Figure 5. Hazard Plot for Overall Survival (Months)

In the survival analysis conducted in this study, four main plots were used to illustrate the distributional characteristics and event risk of the Overall Survival data. The first plot, namely the Probability Plot Fig [2], indicates that the data fit the exponential distribution reasonably well, as shown by their consistency with the fitted reference line and the 95% confidence interval. However, slight deviations are observed in several data points, particularly at the early survival times, which may indicate a minor lack of fit of the model for early observations. The Cumulative Failure Plot Fig [3] illustrates how the proportion of subjects experiencing the event increases over time, although the rate of increase becomes slower over longer periods. This finding suggests that although

the probability of failure continues to increase, the failure rate tends to become more stable at later survival times, which is consistent with the exponential distribution applied in this analysis.

Furthermore, the Survival Plot Fig. [4] shows a substantial decline in survival probability over time. This curve indicates that the longer the survival time, the greater the likelihood that subjects will experience the observed event. Based on the fitted exponential model, the theoretical median survival time was approximately 54.86 months. This value differs from the empirical median survival time of 41.425 months reported in Table 2, as the empirical median is calculated directly from the observed data, whereas the theoretical median is derived from the estimated exponential model parameter $\hat{\theta} = 64.54$. Lastly, the Hazard Plot Fig
cite4 demonstrates that the event risk remains relatively stable throughout the observation period, with a consistent hazard rate of approximately 0.0121-0.0189 per month. This pattern supports the appropriateness of using the exponential distribution, as the risk of experiencing the event does not show substantial fluctuations over time but remains relatively constant. Overall, these four plots provide a comprehensive overview of the distributional pattern and risk characteristics of the survival data analyzed in this study.

Based on the estimated exponential survival function, the probability of surviving up to 60 months was approximately 39.47%. Although this value appears lower than the five-year overall survival rates of approximately 94-95% reported in previous epidemiological studies [6], [11], [13], these estimates are not directly comparable because they were derived from different study populations, sample sizes, follow-up designs, and analytical approaches. The present study was based on a small retrospective cohort from the cBioPortal database and estimated survival using an exponential parametric model under a Type I censoring framework. Therefore, the estimated survival probability should be interpreted as a sample-specific, model-based estimate rather than a population-level measure of five-year survival.

The estimated survival probability at 60 months obtained from the exponential model was approximately S (60) = 0.39. This estimate is significantly lower than the five-year survival rates of approximately 94-95% reported in previous studies [6], [11], [13]. Several factors may explain this discrepancy. First, the present study analyzed only 134 complete observations obtained under Type I censoring, whereas previous studies generally involved substantially larger datasets. Second, differences in patient characteristics, observation periods, and data collection procedures may have influenced the estimated survival probabilities. Third, the exponential distribution assumes a constant hazard rate throughout the observation period, which may not fully capture the underlying survival pattern of testicular cancer patients. Therefore, the estimated survival probability should be interpreted within the context of these assumptions and data limitations. This assumption may be overly restrictive for testicular cancer because the risk of mortality is unlikely to remain constant throughout the disease course. Following diagnosis and treatment, patients may experience changing hazard patterns due to therapeutic response, recurrence, or disease progression. Consequently, the exponential model may oversimplify the underlying survival process and produce survival estimates that differ from observed clinical outcomes.

More flexible parametric models, such as the Weibull or log-normal distributions, allow hazard rates to vary over time and may therefore provide a better representation of the underlying survival dynamics. The Weibull model accommodates monotonically increasing or decreasing hazards through its shape parameter, whereas the log-normal model can represent non-monotonic hazard patterns. Comparing these alternative models using goodness-of-fit criteria such as the Akaike Information Criterion (AIC) would provide a more comprehensive assessment of the most appropriate survival model for testicular cancer data.

4. CONCLUSION

This study applied a Type I censored exponential survival model to testicular cancer patient data obtained from cBioPortal. Among 134 eligible patients, 82 were classified as uncensored and 52 were censored at the 60-month observation limit. The Anderson–Darling goodness-of-fit test yielded a p-value of 0.067, indicating insufficient evidence to reject the exponential distribution assumption. The estimated parameter was $\hat{\theta} = 64.54$ months, corresponding to the model-based mean time to failure (MTTF), with a constant hazard rate of 0.0155 per month. Based on the fitted exponential model, the estimated survival probability at 60 months was approximately 0.39, providing a quantitative description of long-term survival under the model assumptions. These findings demonstrate that the exponential model offers a parsimonious and statistically sound baseline for evaluating survival probability and mortality risk in the analyzed cohort. The observed survival estimates reflect the specific composition and follow-up characteristics of the sample, highlighting the importance of population-specific baseline evaluations. Future studies should consider incorporating relevant clinical covariates, extending follow-up durations, and evaluating alternative multi-parameter or flexible models across broader multicenter datasets to further enhance clinical interpretability.

REFERENCES

[1] J. Chavarriaga, L. Nappi, A. Papachristofilou, C. Conduit, and R. J. Hamilton, "Testicular cancer," *The Lancet*, vol. 406, no. 10498, pp. 76–90, 2025. [Online]. Available: https://doi.org/10.1016/S0140-6736(25)00455-6

[2] J. Huang, S. C. Chan, M. S. Tin *et al.*, "Worldwide distribution, risk factors, and temporal trends of testicular cancer incidence and mortality: A global analysis," *European Urology Oncology*, vol. 5, no. 5, pp. 566–576, 2022. [Online]. Available: https://doi.org/10.1016/j.euo.2022.06.009

[3] S. Giona, "The epidemiology of testicular cancer," in *Urologic Cancers*. Exon Publications, 2022.

[4] H. Sung, J. Ferlay, R. L. Siegel *et al.*, "Global cancer statistics 2020: Globocan estimates of incidence and mortality worldwide for 36 cancers in 185 countries," *CA: A Cancer Journal for Clinicians*, vol. 71, no. 3, pp. 209–249, 2021. [Online]. Available: https://doi.org/10.3322/caac.21660

[5] R. I. H. Chong, J. J. Leow, Z. W. Choo, R. Salada, D. Z. P. Yong, and Y. L. Chong, "Testicular self-examination for early detection of testicular cancer," *World Journal of Urology*, vol. 41, no. 4, pp. 941–951, 2023. [Online]. Available: https://doi.org/10.1007/s00345-023-04381-4

[6] American Cancer Society, "Survival rates for testicular cancer," https://www.cancer.org/cancer/types/testicular-cancer/detection-diagnosis-staging/survival-rates.html, 2025, accessed: 2026-03-28.

[7] D. D. Rivera Rosales and D. A. Tejada, "Fundamentals and applications of survival analysis for health research," *Alerta*, vol. 8, no. 3, pp. 305–314, 2025. [Online]. Available: https://doi.org/10.5377/alerta.v8i3.20675

[8] A. Kurniawan, A. T. Previan, and Z. I. Nurrohmah, "Survival function and hazard function analysis of exponential distribution in type i censored survival data: A case study of breast cancer patients," *Barekeng*, vol. 17, no. 3, pp. 1795–1802, 2023. [Online]. Available: https://doi.org/10.30598/barekengvol17iss3pp1795-1802

[9] S. Kaindal and B. Venkataramana, "A comparative analysis of parametric survival models and machine learning methods in breast cancer prognosis," *Scientific Reports*, vol. 15, p. 153828, 2025. [Online]. Available: https://doi.org/10.1038/s41598-025-15696-0

[10] N. Taketomi, K. Yamamoto, C. Chesneau, and T. Emura, "Parametric distributions for survival and reliability analyses, a review and historical sketch," *Mathematics*, vol. 10, no. 20, p. 3907, 2022. [Online]. Available: https://doi.org/10.3390/math10203907

[11] K. H. Pang, H. M. Alnajjar, and A. Muneer, "Diagnosis, treatment and survival from testicular cancer: Real-world data from the national health service england between 2013 and 2020," *Clinical Genitourinary Cancer*, vol. 23, no. 4, p. 102367, 2025. [Online]. Available: https://doi.org/10.1016/j.clgc.2025.102367

[12] B. J. Mantiri, G. P. Siregar, and S. M. Warli, "5-year relative survival rate between seminoma and non-seminoma testicular cancer in single center," *Indonesian Journal of Urology*, vol. 30, no. 3, 2023.

[13] L. Mangone, F. Marinelli, I. Bisceglia *et al.*, "Incidence and survival of testicular cancers in a province in northern italy and their association with second tumors," *Biology*, vol. 12, no. 11, p. 1409, 2023. [Online]. Available: https://doi.org/10.3390/biology12111409

[14] A. Kurniawan, A. F. Yosifa, V. N. P. Wardhani, and A. Kholidiyah, "Estimation of survival function in head and neck cancer patients using the kaplan–meier method," *J Statistika: Jurnal Ilmiah Teori dan Aplikasi Statistika*, vol. 16, no. 1, pp. 452–461, 2023.

[15] C. A. W. Subekti, I. Nilasari, D. M. Syarif, I. Syahzaqi, and A. Kurniawan, "Survival time analysis of multiple myeloma patients using type i censored exponential distribution parameter estimation," *JTAM: Jurnal Teori dan Aplikasi Matematika*, vol. 9, no. 3, pp. 1082–1095, 2025. [Online]. Available: https://doi.org/10.31764/jtam.v9i3.32179

[16] World Health Organization, "Sdg target 3.4: Noncommunicable diseases and mental health," https://www.who.int/data/gho/data/themes/topics/indicator-groups/indicator-group-details/GHO/sdg-target-3.4-noncommunicable-diseases-and-mental-health, Geneva, Switzerland, 2026.

[17] W. H. Organization, "Cancer," https://www.who.int/news-room/fact-sheets/detail/cancer, Geneva, Switzerland, April 2026.

[18] L. Jantschi and S. D. Bolboacua, "Computation of probability associated with anderson-darling statistic," *Mathematics*, vol. 6, no. 6, 2018. [Online]. Available: https://doi.org/10.3390/math6060088