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APPLICATION OF OPTIMAL CONTROL THEORY TO INVENTORY PROBLEMS THAT ARE INCREASING AT PT. INDUSTRY PLYWOOD TJIPTA RIMBA DJAJA

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ABSTRACT

Every company that carries out businees activities generally has inventory. Inventories include raw materials, semi-finished goods or finished goods. Inventories of goods in the company have increased and decreased. An increase in inventory can cause losses, because the cost of storing and maintaining in the warehouse is too high. While a decrease in inventory can result in a shortage of inventory. The purpose of this research is to determine the level of optimal inventory in PT. Industry Plywood Tjipta Rimba Djaja. Using the optimal control theory model and analyzing the stability of the dynamic differential equation, to find the optimal inventory level. Obtained optimal inventory levels achieve stability at the time $6.306,0684m^3$. For the planning length of 12 months includes: raw material inventory (logs sengon and rambung), production (finished materials in process) and finished plywood or plywood products that are in the warehouse. From this research optimal control theory can be applied in PT. Industry Plywood Tjipta Rimba Djaja to optimize inventory on the problem of increasing inventory.

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1. INTRODUCTION

Every company innovates to create quality products in the face of increasingly fierce global competition between companies, and can build accurate inventory control. One of the industrial companies that produce goods is PT. Industry Plywood Tjipta Rimba Djaja (I China Factory) is a company engaged in the wood industry, where the raw material used is Sengon logs and rambung to produce the main product namely plywood.

Inventory is an important part of the company's production activities, which are consistently obtained, modified and further processed, then sold. With the avaibility of sufficient inventory in the warehouse, aims to facilitate production activities or services to consumers. Supplies of raw logs with sengon logs dan rambung types at PT. Industry Plywood Tjipta Rimba Djaja is very much needed, because the two woods are the main raw material as well as support for the company's production activities to produce plywood. Inventory in maximum quantity will maintain the company's capability to serve every customer order, but it will have an impact on increasing inventory storage and production costs in the company.

Demand is the amount of commodity desired by the market at a certain price level, certain income level and a certain period of time. Periodically, consumer demand for an item is influenced by the price of the goods needed, income levels, the number of residents in the area, future interests and predictions, and prices of other commodities. Demand for plywood at PT. Industry Plywood Tjipta Rimba Djaja comes from within and outside the country. Demand is experiencing instability from each consumer with a different quantity every month.

One way that can be done to overcome the problem of increasing inventory is to optimize the existing inventory, and to optimize inventory optimal control theory can be used. Optimal control theory is a branch of mathematics, which aims to choose the optimal way to control a dynamic systems so that it has a certain value. Modeling the optimal inventory problem based on the inventory system, can use dynamic differential equations and objective functions to obtain differential equations for inventory when there is an increase. Therefore, the inventory problem can also be chosen as an optimal form of control model.

Research that has existed to date, some of which also axamine the application of control theory to inventory. Such as, previous research conducted by Usvita and Andiraja (2017) "Optimal Control on the Problem of Increasing Inventory", the optimal control model is obtained with dynamic differential equations and objective functions for the problem of increasing inventory, then stability analysis is carried out to find the optimal inventory level.

2. RESEARCH METHODE

2.1 Research Sources And Variables

The data used as secondary data sources and research variables was obtained from PT. Industry Plywood Tjipta Rimba Djaja, in the form of documentation of inventory stock data, production data, demand data, inventory storage cost data, and plywood production cost data. The data collection period is Mei 2020 to April 2021.

2.2 Data Analysis Techniques

The research was carried out with a procedure described in nine stages, starting with the initial step namely identifying the problem and ending after finding the following conclusions and suggestions:

1. Collecting inventory stock data, production data, demand data, inventory storage cost data, and plywood production cost data period is Mei 2020 to April 2021.
2. Establish a dynamic differential equation model for an increasing product inventory model.

$$\dot{S} = C(t) + u(t)S(t) \qquad t \in [0, t_1] \tag{2.1}$$

With $u(t) = n(t) - \alpha(t)$

Description:

$S(t)$	=	Inventory level at time t
$C(t)$	=	Average production at time t
S_0	=	Initial inventory level at time t
$n(t)$	=	Average increase function at time t
$\alpha(t)$	=	Average slump at time t
$u(t)$	=	The difference between the average function of increase and decrease

3. Establish an objective function model

$$J = \frac{1}{2}\int_0^{t} \left\{ a\left[S(t) - \hat{S}\right]^2 + b\left[C(t) - \hat{C}\right]^2 \right\} dt \tag{2.2}$$

Description:

$\hat{S}$	=	Target inventory level at time t
$\hat{C}$	=	Target production level at time t
$a(t)$	=	Average storage cost at time t
$b(t)$	=	Average production cost at time t

4. Defined by Hamilton's equation, for an increasing product inventory model

$$H=-\frac{1}{2}\left[a\left(S-\hat{S}\right)^2+b\left(C-\hat{C}\right)^2\right]+\lambda y \text{ with } y=C+uS \quad (2.3)$$

5. Defined by Lagrange's equation, for an increasing product inventory model

$$L=-\frac{1}{2}\left[a\left(S-\hat{S}\right)^2+b\left(C-\hat{C}\right)^2\right]+\left(\lambda+\mu\right)y \text{ with } y=C+uS \quad (2.4)$$

6. Prove the optimal conditions of the Hamilton dan Lagrange equation
   a) $H_C=0$ (2.5)
   b) $L_S=-\dot{\lambda}$ (2.6)
   c) $L_C=0$ (2.7)
   d) $\mu\geq 0;\mu y\geq 0$ (2.8)

7. From proving the optimal conditions, it is assumed in two cases in the form of an explicit solution:
   a) The function U is a constant
   b) The function $\frac{a}{b}+\dot{u}+u^2$ is a constant

8. Perform a stability analysis, stability analysis is measured through the completion of an explicit form to find the optimal inventory level.

$$S\left(t\right)=F_{11}e^{rt}+F_{12}e^{-rt}+Q_1\left(t\right) \qquad t\in\left[0,T\right] \quad (2.9)$$

9. Make conclusions and suggestions.

3. RESULT AND ANALYSIS

3.1 Description of Data

The data collection in this study is divided into several data, starting from May 2020 to April 2021, it is known as follows:

1) $\left(S(12)\right)$ = $13.005,32m^3$
2) $\left(S_0\right)$ = $832,23m^3$
3) $\left(a\left(S(12)\right)\right)$ = Rp. 15.001.030
4) $\left(C(12)\right)$ = $532,32m^3$
5) $\left(C_0\right)$ = $418,48m^3$
6) $\left(\hat{C}\right)$ = $198,72m^3$
7) $\left(b\left(C(12)\right)\right)$ = Rp. 600.527.816

3.2 Optimal Control Model on Inventory

This study discusses the optimal control theory on the problem of increasing inventory, carried out with the following steps:

1. Form the following dynamic differential equation model:
$$\dot{S} = C(t) + u(t)S(t) \qquad t \in [0, t_1] \tag{3.1}$$

2. The form of the objective function model is as follows:
$$J = \frac{1}{2}\int_0^t \left\{ a\left[S(t) - \hat{S}\right]^2 + b\left[C(t) - \hat{C}\right]^2 \right\} dt \tag{3.2}$$

Equation (3.1) and (3.2) are non-negative constraints.

3. Defined by Hamilton's equation, based on equation (3.1) and (3.2) it is :
$$H = -\frac{1}{2}\left[a\left(S - \hat{S}\right)^2 + b\left(C - \hat{C}\right)^2 \right] + \lambda y \quad \text{with } y = C + uS \tag{3.3}$$

4. Define by the following Lagrange equation:
$$L = -\frac{1}{2}\left[a\left(S - \hat{S}\right)^2 + b\left(C - \hat{C}\right)^2 \right] + (\lambda + \mu) y \quad \text{with } y = C + uS \tag{3.4}$$

5. Prove the optimal conditions formed from the Hamilton and Lagrange equations, as follow:
   a) First Condition:
   $$H_C = 0 \tag{3.5}$$
   b) Second Condition:
   $$L_S = -\dot{\lambda} \tag{3.6}$$
   c) Third Condition:
   $$L_C = 0 \tag{3.7}$$
   d) Fourth Condition:
   $$\mu \geq 0; \mu y \geq 0 \tag{3.8}$$

Equation (3.8) implies $\mu = 0$ based on equation (2.1) and equation (3.5) it is obtained:
$$\dot{S} = \left(\hat{C} + \frac{\lambda}{b} \right) + uS \tag{3.9}$$

With the derivative of the equation (3.9) it is obtained:
$$\ddot{S} = \frac{\dot{\lambda}}{b} + \dot{u}S + u\dot{S} \tag{3.10}$$

By substituting the equation (3.6) and (3.9) into equation (3.10) it is obtained:
$$\ddot{S} = \frac{\left(a\left(S - \hat{S}\right) - (\lambda + \mu)u \right)}{b} + \dot{u}S + u\left(\left(\hat{C} + \frac{\lambda}{b} \right) + uS \right) \tag{3.11}$$

And based on equation (3.7) , then $b\left(C - \hat{C}\right) = (\lambda + \mu)$, so that:
$$\ddot{S} = \frac{\left(a\left(S - \hat{S}\right) - (\lambda + \mu)u \right)}{b} + \dot{u}S + u\left(\hat{C} + \frac{\lambda}{b} + uS \right) \tag{3.12}$$

Based on equation (3.5) then, $C - \hat{C} = \frac{\lambda}{b}$, so that:
$$\ddot{S} - \left(\frac{a}{b} + \dot{u} + u^2 \right) S = -\frac{a\hat{S}}{b} + u\hat{C} \tag{3.13}$$

6. From proving the optimal conditions, it is assumed in two cases in the form of an explicit solution.
   a) The function U is a constant.

If the U function is a constant, it can be seen in the differential equation (3.13) so that the following equation is formed:

$$\ddot{S}-\left(\frac{a}{b}+u^2\right)S=-\frac{a\hat{S}}{b}+u\hat{C} \qquad (3.14)$$

Equation (3.14) is a second order differential equation which is not homogeneous. The first step that must be taken to solve equation (3.14) is to choose a general solution to the homogenous equation, so that the following characteristic equation is formed:

$$r^2-\left(\frac{a}{b}+u^2\right)=0 \qquad (3.15)$$

From equation (3.15) , the solution of the differential equation is obtained with different real roots. So that the following equation is formed:

$$r_1=\sqrt{\left(\frac{a}{b}+u^2\right)}=r \qquad r_2=-\sqrt{\left(\frac{a}{b}+u^2\right)}=-r$$

r = 475 -r = -475

For solution (3.15) produces the following equation to analyze the stability of inventory levels:

$$S(t)=F_{11}e^{rt}+F_{12}e^{-rt}+Q_1(t) \qquad t\in[0,T] \qquad (3.16)$$

= 6.306,0684

Where $Q_1(t)$ is an additional solution to the non-homogeneous equation (3.14) then we get:

$$Q_1(t)=\frac{a\hat{S}-uaC}{a+bu^2}$$

= 13,6784

Then determine $\left(C(t)\right)$ from the conditions $S(0)=S_0$ and $S(t_1)=N$ in equation (3.16), so that the following equation is formed:

a. For $t=0$ obtained: $S_0=F_{11}(1)+F_{12}(1)+Q_1(0)$
b. For $t=t_1$ obtained: $N=F_{11}e^{rt_1}+F_{12}e^{-rt_1}+Q_1(t_1)$

Value F_{11} and F_{12} can be solved as follows:

$$F_{11}=\frac{e^{-rt_1}\left(S_0-Q_1(0)\right)-\left(N-Q_1(t_1)\right)}{e^{-rt_1}-e^{rt_1}} \qquad F_{12}=\frac{-e^{rt_1}\left(S_0-Q_1(0)\right)+\left(N-Q_1(t_1)\right)}{e^{-rt_1}-e^{rt_1}}$$

= 159,0384 = 1.166,1551

Based on equations (3.9) and (3.16) obtained as follows:

$$\lambda=a\left(\dot{S}-\hat{C}-uS\right)$$

$$\lambda=a\left(F_{11}(r-u)e^{rt}-F_{12}(r+u)e^{-rt}+\dot{Q}_1-\hat{C}-uQ_1\right) \qquad (3.17)$$

From equation (3.9) substituted into equation (3.17) it is obtained as follows:

$$C(t)=\hat{C}+\left(F_{11}(r-u)e^{rt}-F_{12}(r+u)e^{-rt}+\dot{Q}_1(t)-\hat{C}-u\left(Q_1(t)\right)\right) \qquad (3.18)$$

b) The function $\frac{a}{b}+\dot{u}+u^2$ is a constant

Assumed $\frac{a}{b}+\dot{u}+u^2=k_1^2$ (3.19)

So, that the equation (3.14) is obtained:

$$\ddot{S}-\left(k_1^2\right)S=-\frac{a\hat{S}}{b}+u\hat{C} \qquad t\in\left[0,t_1\right] \tag{3.20}$$

Substituting $u(t)$ into equation (3.19) we get:

$$\ddot{S}-\left(k_1^2\right)S=\frac{-a\hat{S}}{b}+\frac{z\left(e^{2zt}+1\right)}{\left(e^{2zt}-1\right)}\hat{C} \tag{3.21}$$

Equation (3.21) is a second-order differential equation that is not homogeneous, the first step to solving equation (3.21) is to determine the general solution to the homogeneous equation. So, that the following characteristic equation will be formed:

$$r^2-\left(k_1^2\right)=0 \tag{3.22}$$

So that the roots of the equation are obtained, namely $r_1=k_1$ and $r_2=-k_1$ to solve the following equation (3.22) :

$$S(t)=F_{11}e^{k_1t}+F_{12}e^{-k_1t}+Q(t) \tag{3.23}$$

Where $Q(t)$ is the solution of the homogeneous equation of equation (3.21), so we get:

$$Q(t)=\frac{a\hat{S}}{bk_1^2}+u_1e^{k_1t}+u_2e^{-k_1t}$$

With u_1 and u_2 is the antiderivative of u_1 and u_2.

Furthermore, to determine $\left(C(t)\right)$ using the condition $S(0)=S_0$ and $S(t_1)=N$ the following equation will be obtained:

a. For $t=0$ obtained $S_0=F_{11}+F_{12}+\frac{a\hat{S}}{bk_1^2}$

b. For $t=t_1$ obtained $N=\left(F_{11}+u_1\left(t_1\right)\right)e^{k_1t_1}+\left(F_{12}u_2\left(t_1\right)\right)e^{-k_1t_1}+\frac{a\hat{S}}{bk_1^2}$

Value F_{11} and F_{12} can be solved as follows:

$$F_{11}=\frac{e^{-k_1t_1}\left(S_0-\frac{a\hat{S}}{bk_1^2}\right)-\left(N\left(u_1\left(t_1\right)e^{k_1t_1}+u_2\left(t_1\right)e^{-k_1t_1}+\frac{a\hat{S}}{bk_1^2}\right)\right)}{e^{-k_1t_1}-e^{k_1t_1}} \qquad F_{12}=\frac{-e^{k_1t_1}\left(S_0-\frac{a\hat{S}}{bk_1^2}\right)+\left(N\left(u_1\left(t_1\right)e^{k_1t_1}+u_2\left(t_1\right)e^{-k_1t_1}+\frac{a\hat{S}}{bk_1^2}\right)\right)}{e^{-k_1t_1}-e^{k_1t_1}}$$

Based on equation (3.9) and equation (3.23) obtained as follows:

$$\lambda=b\left(\dot{S}-\hat{C}-uS\right)$$

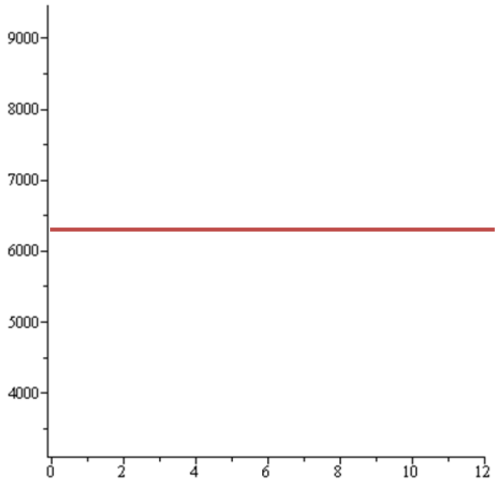
So that from equation (3.23) can be substituted into equation (3.5) and we get:

$$C(t)=\left(\left(k_1-u(t)\right)\left[\left(F_{11}+u_1(t)\right)\right]e^{k_1t}-\left(k_1+u(t)\right)\left[\left(F_{12}+u_2(t)\right)\right]e^{-k_1t}\right)-\frac{a\hat{S}}{bk_1^2}u(t) \tag{3.24}$$

7. Stability Analysis

Stability analysis was carried out to find the optimal inventory level from equation (3.17) for $t\in\left[0,12\right]$ using maple software.

Presented in the following graphic image:



Pict.1. Graphic $S(t)$ for $t \to 12$

Based on pict.1, it can be seen that for $t \to 12$ the inventory level value is $\left(S(t)\right) \to 6.306{,}0684m^3$ or $\left(S(t)\right) \to N$ it can be concluded that $S(t)$ is stable because $S(t)$ for $t \to 12$ goes to the value with the maximum inventory level (N).

4. CONCLUSION

Based on the result of the research that has been done, the optimal inventory level is obtained at $6.306{,}0684m^3$ for the planning length of 12 months. The inventories include: Raw materials sengon logs and rambung, plywood products in process, and finished plywood products ready for sale. It is concluded that Optimal Control Theory can be applied at PT. Industry Plywood Tjipta Rimba Djaja.
Other researchers who are interested in conducting research with optimal control theory models, are expected to develop and modify dynamic differential equation models. So that the resulting mod can be applied to other larger companies in other fields.

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