



# APPLICATION OF THE FORWARD RECURSIVE EQUATION MODEL ON WASTE TRANSPORTATION ROUTES IN MEDAN JOHOR SUB-DISTRICT USING DYNAMIC PROGRAMMING ALGORITHM

Maudya Nur Azura<sup>1</sup>, Dinda Permata Sari<sup>2</sup>, Syarifah Hanum<sup>3</sup>, Ismail Husein<sup>4</sup>

<sup>1,2,3,4</sup>Department of Mathematics, Universitas Islam Negeri Sumatera Utara, Medan, Indonesia

## Article Info

### Article history:

Received 01 08, 2024  
Revised 01 15, 2024  
Accepted 01 22, 2024

### Keywords:

Routes, Dynamic Programming,  
Forward Recursive Equations

## ABSTRACT

Waste problems often occur in big cities such as Jakarta, Surabaya, Medan, Bandung, Yogyakarta and Semarang. Urban waste is waste produced by human activities in urban areas, including various types of materials such as plastic, paper, metal and organic materials. Therefore, waste needs to be collected, transported and disposed of immediately to prevent negative impacts on the environment. With the increase in population in urban areas, it is likely that the amount of waste will also increase. The solution that can be used to solve this problem is to optimize the distance using a dynamic programming model forward recursive equation algorithm. The application of dynamic programming can be used to solve various problems such as determining the shortest distance, optimal flow, task or resource scheduling, production planning, network optimization, and problems in research and development projects. Based on determining the optimal route for transporting waste in Medan Johor District using the dynamic programming algorithm forward recursive equation model, the total distance is 26,2km.

*This is an open access article under the [CC BY-SA](https://creativecommons.org/licenses/by-sa/4.0/) license.*



## Corresponding Author:

Maudya Nur Azura,  
Department Of Mathematic,  
Universitas Islam Negeri Sumatera Utara Medan  
Email: [maudya0703201003@uinsu.ac.id](mailto:maudya0703201003@uinsu.ac.id)

## 1. INTRODUCTION

The community service carried out by students is directed through the Real Work Lecture process which conveys knowledge at the university to the location of Government Agency Service. In this case students participate in support government as part from the Practical Work Lecture placement. The location of the Student Practical Work Lecture activities is placed at the agency for students to practice, namely the Medan City Environmental Service. Practical work in the environment can give students direct experience in this field, as well as assist environmental services in carrying out certain projects or research [1]. Students can be involved in environmental surveys, data analysis, or even policy development related to the environment [2]. Problem rubbish often happens in cities big such as Jakarta, Surabaya, Medan, Bandung, Yogyakarta and Semarang. Enhancement amount rubbish This caused by growth increasing population increasing [3]. Rubbish urban is waste produced by activity humans in the area urban, includes

various types of materials such as plastic, paper, metal, and material organic. By Because that 's rubbish need quick collected, transported, and thrown away for prevent impact negative to environment [4]. With increase amount urban residents, perhaps big amount rubbish Also increase.

Dynamic programs, too known as dynamic programming, generally used for handle problem optimization where there is various solution with optimal value (minimum or maximum). Achieved solution the optimal value called as optimal solution for problem that, remember possible there is a number of possible solution reach the optimal value [5]. Term dynamic programming appear because his approach tends analyze and take notes results calculation on every step 20 through use table, so solution calculation can understand in a way detailed. Application dynamic programming can used in finish various problem like determine distance shortest, optimal flow, scheduling task or source power, planning production, optimization network, and problem on project study and development. For handle various problem these, various dynamic programming method can applied, as appropriate with characteristics different optimizations on each problem [6]. Characteristics general from possible problems resolved with a dynamic program [7]:

- a. Problem can solved become a number of stage (stage), where more subproblems small and optimally, that can used for solve problem main.
- b. On every stage only take One optimal decision.
- c. Between results decision on stage one with stage next own linkages.
- d. Results decision on every stage changed from related situations become situation next on stage next.
- e. Systematics solution made for get results or optimal decisions throughout stages.
- f. Results optimal decisions are taken on stage final.

There is connection recursive identifying optimal results for every condition on stage beginning And give optimal results for every condition on stage next.

## 2. RESEARCH METHODE

To solve this problem the author uses a quantitative descriptive method, namely a form of research that focuses on the facts and characteristics of the research object by connecting the variables involved. Quantitative research often focuses on using data in the form of numbers, such as collecting, interpreting, and presenting research results. Meanwhile, descriptive research is used to explain or describe existing phenomena, including natural and man-made phenomena [8]. The selection of this type of research is based on research objectives and distance data between waste collection points, which can be used with dynamic programming and forward recursive equations. This will be material for analyzing the shortest route to take when doing so.

## 3. RESULT AND ANALYSIS

Following This is location data or point transportation waste in Medan Johor District:

Table 1. Village Names and Collection Points

No	Sub-District	Symbols	Transportation Point
1	Pangkalan Masyhur	a	Karya Kasih Street
2	Kedai Durian	b	Brigjen. Zein Hamid Street
3	Kwala Bekala	c	Ngumban Surbakti Street
4	Suka Maju	d	SM Raja Street
5	Gedung Johor	e	Karya Jaya Street
6	Titi Kuning	f	Abdul Haris Nasution Street

The distance between one transportation point to another in km is as follows:

1. Pangkalan Masyhur (a) to Pangkalan Masyhur (a) = 0 km
2. Pangkalan Masyhur (a) to Kedai Durian (b) = 4.2 km
3. Pangkalan Masyhur (a) to Kwala Bekala (c) = 4.7 km
4. Pangkalan Masyhur (a) to Suka Maju (d) = 7.2 km
5. Pangkalan Masyhur (a) to Gedung Johor (e) = 3.4 km
6. Pangkalan Masyhur (a) to Titi Kuning (f) = 2.9 km
7. Kedai Durian (b) to Kedai Durian (b) = 0 km
8. Kedai Durian (b) to Kwala Bekala (c) = 7.6 km

9. Kedai Durian (b) to Suka Maju (d) = 4.3 km
10. Kedai Durian (b) to Gedung Johor (e) = 2.8 km
11. Kedai Durian (b) to Titi Kuning (f) = 5.9 km
12. Kwala Bekala (c) to Kwala Bekala (c) = 0 km
13. Kwala Bekala (c) to Suka Maju (d) = 7.3 km
14. Kwala Bekala (c) to Gedung Johor (e) = 7.3 km
15. Kwala Bekala (c) to Titi Kuning (f) = 1.7 km
16. Suka Maju (d) to Suka Maju (d) = 0 km
17. Suka Maju (d) to Gedung Johor (e) = 6.6 km
18. Suka Maju (d) to Titi Kuning (f) = 6.4 km
19. Gedung Johor (e) to Gedung Johor (e) = 0 km
20. Gedung Johor (e) to Titi Kuning (f) = 5.3 km

Table 2 Distance Between Transportation Points in Km

	a	b	c	d	e	f
a	0	4,2	4,7	7,2	3,4	2,9
b	4,2	0	7,6	4,3	2,8	5,9
c	4,7	7,6	0	7,3	7,3	1,7
d	7,2	4,3	7,3	0	6,6	6,4
e	3,4	2,8	7,3	6,6	0	5,3
f	2,9	5,9	1,7	6,4	5,3	0

**Calculation Using Dynamic Programming**

In this dynamic programming calculation, the process moves forward, with the optimal result determined from stage 1 and ending at stage n.

**Phase I:**

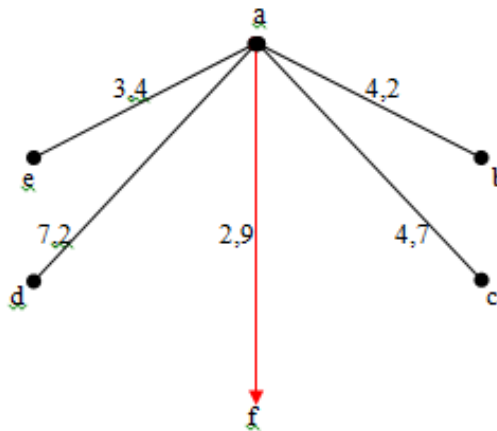


Figure 1 Route Graph with Dynamic Programming Phase I

$$F(a, i) = c_{i,1}$$

With:

- $i$  : Name of urban village or transportation point (stage)
- $f(a, i)$  : Optimal value function
- $c_{i,1}$  : Output value at stage  $i$

So the result is obtained:

$$f(a, b) = 4,2 \qquad f(a, d) = 7,2 \qquad f(a, f) = 2,9$$

$$f(a, c) = 4,7 \qquad f(a, e) = 3,4$$

The first stage produces 5 alternative roads that can be passed, with a as the status. Then the feasible solution is **Titi Kuning (f)**.

### Phase II:

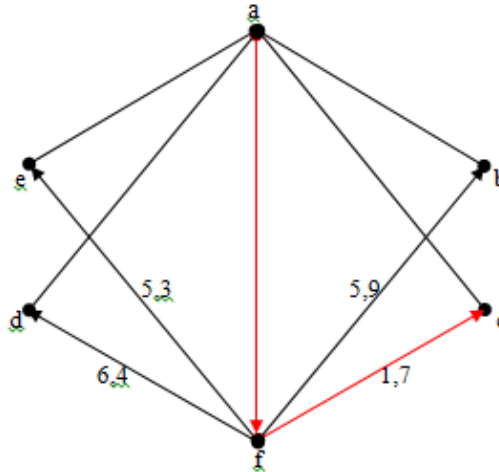


Figure 2 Route Graph with Dynamic Programming Phase II

$$F(S, i) = \min_{j \in S} \{C_{f,i} + F(j, S - \{j\})\} \text{ for } |S| = 1$$

With:

- $S$  = State/some conditions that may occur at each stage of the *dynamic programming* problem.  
 $|S|$  = Because the optimal function for each stage is only performed in one state/condition with the status is b,c,d,e,f (1).

So, the result is obtained:

$$\begin{aligned} f(b, c) &= \min C_{bc} + f(a, c) = 7,6 + 4,7 = 12,3 \\ f(b, d) &= \min C_{bd} + f(a, d) = 4,3 + 7,2 = 11,5 \\ f(b, e) &= \min C_{be} + f(a, e) = 2,8 + 3,4 = 6,2 \\ f(b, f) &= \min C_{bf} + f(a, f) = 5,9 + 2,9 = 8,8 \\ f(c, b) &= \min C_{cb} + f(a, b) = 7,6 + 4,2 = 11,8 \\ f(c, d) &= \min C_{cd} + f(a, d) = 7,3 + 7,2 = 14,5 \\ f(c, e) &= \min C_{ce} + f(a, e) = 7,3 + 3,4 = 10,7 \\ \mathbf{f(c, f)} &= \mathbf{\min C_{cf} + f(a, f) = 1,7 + 2,9 = 4,6} \\ f(d, b) &= \min C_{db} + f(a, b) = 4,3 + 4,2 = 8,5 \\ f(d, c) &= \min C_{dc} + f(a, c) = 7,3 + 4,7 = 12 \\ f(d, e) &= \min C_{de} + f(a, e) = 6,6 + 3,4 = 10 \\ f(d, f) &= \min C_{df} + f(a, f) = 6,4 + 2,9 = 9,3 \\ f(e, b) &= \min C_{eb} + f(a, b) = 2,8 + 4,2 = 7 \\ f(e, c) &= \min C_{ec} + f(a, c) = 7,3 + 4,7 = 12 \\ f(e, d) &= \min C_{ed} + f(a, d) = 6,6 + 7,2 = 13,8 \\ f(e, f) &= \min C_{ef} + f(a, f) = 5,3 + 2,9 = 8,2 \\ f(f, b) &= \min C_{fb} + f(a, b) = 5,9 + 4,2 = 10,1 \\ f(f, c) &= \min C_{fc} + f(a, c) = 1,7 + 4,7 = 6,4 \\ f(f, d) &= \min C_{fd} + f(a, d) = 6,4 + 7,2 = 13,6 \\ f(f, e) &= \min C_{fe} + f(a, e) = 5,3 + 3,4 = 8,7 \end{aligned}$$

The second stage produces 20 alternative roads that can be passed with (b, c, d, e, f). Then the feasible solution is **Kwala Bekala (c)**.

### Phase III:

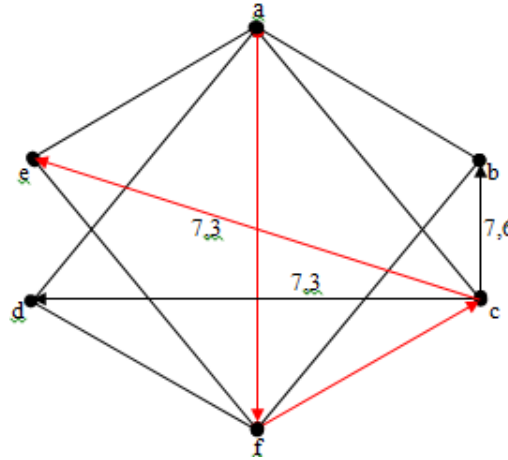


Figure 3 Route Graph with Dynamic Programming Phase III

$$F(S, i) = \min_{j \in S} \{c_{f,i} + F(j, S - \{j\})\}, \text{ for } |S| = 2 \text{ and } i \neq a, a \notin S$$

$|S| = 2$ , This is because the optimal function for each stage is only performed in two states/conditions with the states being b, c, d, e, f.

So, the result is obtained:

$$f(b, \{c, d\}) = \min[C_{bc} + f(c, d); C_{bd} + f(d, c)] \\ = \min [7,6 + 16,2; 4,3 + 12] = 16,3$$

$$f(b, \{c, e\}) = \min[C_{bc} + f(c, e); C_{be} + f(e, c)] \\ = \min [7,6 + 10,7; 2,8 + 12] = 14,8$$

$$f(b, \{c, f\}) = \min[C_{bc} + f(c, f); C_{bf} + f(f, c)] \\ = \min [7,6 + 4,6; 5,9 + 6,4] = 12,2$$

$$f(b, \{d, e\}) = \min[C_{bd} + f(d, e); C_{be} + f(e, d)] \\ = \min [4,3 + 10; 2,8 + 13,8] = 14,3$$

$$f(b, \{d, f\}) = \min[C_{bd} + f(d, f); C_{bf} + f(f, d)] \\ = \min [4,3 + 9,3; 5,9 + 13,6] = 13,6$$

$$f(b, \{e, f\}) = \min[C_{be} + f(e, f); C_{bf} + f(f, e)] \\ = \min [2,8 + 8,2; 5,9 + 8,7] = 11$$

$$f(c, \{b, d\}) = \min[C_{cb} + f(b, d); C_{cd} + f(d, b)] \\ = \min [7,6 + 11,5; 7,3 + 8,5] = 15,8$$

$$f(c, \{b, e\}) = \min[C_{cb} + f(b, e); C_{ce} + f(e, b)] \\ = \min [7,6 + 6,2; 7,3 + 7] = 13,8$$

$$f(c, \{b, f\}) = \min[C_{cb} + f(b, f); C_{cf} + f(f, b)] \\ = \min [7,6 + 8,8; 1,7 + 10,1] = 11,8$$

$$f(c, \{d, e\}) = \min[C_{cd} + f(d, e); C_{ce} + f(e, d)] \\ = \min [7,3 + 10; 7,3 + 13,8] = 17,3$$

$$f(c, \{d, f\}) = \min[C_{cd} + f(d, f); C_{cf} + f(f, d)] \\ = \min [7,3 + 9,3; 1,7 + 13,6] = 15,3$$

$$f(c, \{e, f\}) = \min[C_{ce} + f(e, f); C_{cf} + f(f, e)] \\ = \min [7,3 + 8,2; 1,7 + 8,7] = 10,4$$

$$f(d, \{b, c\}) = \min[C_{db} + f(b, c); C_{dc} + f(c, b)]$$

$$\begin{aligned}
&= \min [4,3 + 12,3; 7,3 + 11,8] = 16,6 \\
f(d, \{b, e\}) &= \min [C_{db} + f(b, e); C_{de} + f(e, b)] \\
&= \min [4,3 + 6,2; 6,6 + 7] = 10,5 \\
f(d, \{b, f\}) &= \min [C_{db} + f(b, f); C_{df} + f(f, b)] \\
&= \min [4,3 + 8,8; 6,4 + 10,1] = 13,1 \\
f(d, \{c, e\}) &= \min [C_{dc} + f(c, e); C_{de} + f(e, c)] \\
&= \min [7,3 + 10,7; 6,6 + 12] = 18 \\
f(d, \{c, f\}) &= \min [C_{dc} + f(c, f); C_{df} + f(f, c)] \\
&= \min [7,3 + 4,6; 6,4 + 6,4] = 11,9 \\
f(d, \{e, f\}) &= \min [C_{de} + f(e, f); C_{df} + f(f, e)] \\
&= \min [6,6 + 8,2; 6,4 + 8,7] = 14,8 \\
f(e, \{b, c\}) &= \min [C_{eb} + f(b, c); C_{ec} + f(c, b)] \\
&= \min [2,8 + 12,3; 7,3 + 11,8] = 15,1 \\
f(e, \{b, d\}) &= \min [C_{eb} + f(b, d); C_{ed} + f(d, b)] \\
&= \min [2,8 + 11,5; 6,6 + 8,5] = 14,3 \\
f(e, \{b, f\}) &= \min [C_{eb} + f(b, f); C_{ef} + f(f, b)] \\
&= \min [2,8 + 8,8; 5,3 + 10,1] = 11,6 \\
f(e, \{c, d\}) &= \min [C_{ec} + f(c, d); C_{ed} + f(d, c)] \\
&= \min [7,3 + 14,5; 6,6 + 12] = 18,6 \\
f(e, \{c, f\}) &= \min [C_{ec} + f(c, f); C_{ef} + f(f, c)] \\
&= \min [7,3 + 4,6; 5,3 + 6,4] = 11,9 \\
f(e, \{d, f\}) &= \min [C_{ed} + f(d, f); C_{ef} + f(f, d)] \\
&= \min [6,6 + 9,3; 5,3 + 13,6] = 15,9 \\
f(f, \{b, c\}) &= \min [C_{fb} + f(b, c); C_{fc} + f(c, b)] \\
&= \min [5,9 + 12,3; 1,7 + 11,8] = 13,5 \\
f(f, \{b, d\}) &= \min [C_{fb} + f(b, d); C_{fd} + f(d, b)] \\
&= \min [5,9 + 11,5; 6,4 + 8,5] = 14,9 \\
f(f, \{b, e\}) &= \min [C_{fb} + f(b, e); C_{fe} + f(e, b)] \\
&= \min [5,9 + 6,2; 5,3 + 7] = 12,1 \\
f(f, \{c, d\}) &= \min [C_{fc} + f(c, d); C_{fd} + f(d, c)] \\
&= \min [1,7 + 14,5; 6,4 + 12] = 16,2 \\
f(f, \{c, e\}) &= \min [C_{fc} + f(c, e); C_{fe} + f(e, c)] \\
&= \min [1,7 + 10,7; 5,3 + 12] = 12,4 \\
f(f, \{d, e\}) &= \min [C_{fd} + f(d, e); C_{fe} + f(e, d)] \\
&= \min [6,4 + 10; 5,3 + 13,8] = 16,4
\end{aligned}$$

The third stage produces 30 alternative roads that can be passed with (b, c, d, e, f) as the status (see attachment). Then the feasible solution is **Gedung Johor (e)**.

After the calculation is carried out until the last stage, namely stage VI, it is obtained that the total minimum distance of waste transportation in Medan Johor Sub-district starting from Pangkalan Masyhur Village Jl. Karya Kasih and ending at Pangkalan Masyhur Village Jl. Karya Kasih using *dynamic programming* manually is 26.2 km. So the optimal route is a→f→c→e→b→d→a, which starts from Pangkalan Masyhur → Titi Kuning → Kwala Bekala → Gedung Johor → Kedai Durian → Suka Maju → Pangkalan Masyhur.

#### 4. CONCLUSION

Based on the discussion above, it can be concluded that the determination of the optimal route for waste transportation in Medan Johor Sub-district using the *dynamic programming* algorithm *forward recursive equation* model starts from Pangkalan Masyhur → Titi Kuning → Kwala Bekala → Gedung Johor →

Kedai Durian →Suka Maju →Pangkalan Masyhur village. With a total minimum distance traveled of 26.2 km.

## 5. REFERENCES

- [1] N. I. Wijaya, “Efektifitas Program Magang Mahasiswa Bersertifikasi (PMMB) Dalam Mendukung Tujuan Mata Kuliah Kerja Praktik (KP) di Universitas Hang Tuah,” *Proceeding Indones. Carr. Cent. Netw.*, vol. 1, no. 1, pp. 82–89, 2019.
- [2] C. Suharyanti, W. Murtini, and T. Susilowati, “Pengaruh Proses Pembelajaran dan Program Kerja Praktek Terhadap Pengembangan Soft Skills Mahasiswa,” *J. Pendidik. Adm. Perkantoran Univ. Sebel. Maret*, vol. 1, no. 1, pp. 50–62, 2015.
- [3] D. Ernawati, S. Budiastuti, and M. Masykuri, “Komposisi, Jumlah dan Pengembangan Strategi Pengelolaan Sampah di Wilayah Pemerintah Kota Semarang Berbasis Analisis SWOT,” *J. EKOSAINS*, vol. IV, no. 2, pp. 13–22, 2012.
- [4] R. Hariningrum and S. B. Utomo, “Pemanfaatan Limbah Sampah Sebagai Energi Alternatif Pembangkit Tenaga Listrik di Semarang,” *Mar. Sci. Technol. J.*, vol. 1, no. 1, pp. 30–37, 2020.
- [5] T. H. Cormen, C. E. Leiserson, R. L. Rivest, and C. Stein, *Introduction to Algorithms, Fourth Edition*, Fourth. London: The MIT Press, 2022.
- [6] R. P. Sinaga and F. Marpaung, “Perbandingan Algoritma Cheapest Insertion Heuristic dan Nearest Neighbor Dalam Menyelesaikan Traveling Salesman Problem,” *J. Ris. Rumpun Mat. Dan Ilmu Pengetah. Alam*, vol. 2, no. 2, pp. 238–247, 2023, doi: 10.55606/jurrimipa.v2i2.1614.
- [7] N. I. Sari, F. Rakhmawati, and H. Cipta, “Penentuan Rute Terpendek Pendistribusian Produk Kue dengan Menggunakan Algoritma Dynamic Programming Pada Pabrik Kue Ima Brownies,” *J. Marit. Educ.*, vol. 3, no. 1, pp. 207–210, 2021.
- [8] A. Rachman, E. Yochaman, A. ilham Samanlangi, and H. Purnomo, *Metode Penelitian Kuantitatif, Kualitatif dan R&D*, Pertama., no. Januari. Karawang: Saba Jaya Publisher, 2024.